Exam Stochastic Processes for Finance and Derivatives Markets

December 18, 2020

PLEASE WRITE VERY CLEARLY.

Failure to write clearly will be considered as failing the question.

Problem 1 (Futures and forwards). 20 points.

Storage costs of crude oil often depend on how far the storage period is in the future (this happens, for example, when current storages are nearly full). To store 1 barrel of oil for the first year will cost you 4 USD per barrel per year but storing it for the next year – only 3 USD per barrel per year. All storage costs are payable at the beginning of the storage period.

The current price of oil is 45 USD/barrel. Assume that the zero discount curve in US is flat and that the discount rate is 2% p/a.

a) What is the forward price of 1M barrels of oil for forward contract expiring 1.5 years from now? Suppose you purchase this forward contract now (with that forward price written in it). What is the value of this contract now?

b) A year from now, oil price went up to 55 USD/barrel and all storage costs went down to 2 USD per barrel per year. What is now the forward price and the value of the forward contract that you bought in a)?

c) Go back to the situation in a), so assume you are at the point $t=0$ and there is still 1.5 years until the expiry of the forward contract. You are a European oil consumer, so you want not only to hedge your oil price exposure but also your FX exposure. In other words, you would like to enter a forward contract for 1M barrels of oil ***in Euros***. Assume that the current EUR/USD exchange rate is 1.2 (USD for 1 EUR), discount rate is 2% in US as before and discount rate in EU is also flat at -0.5% p/a. All rates are with continuous compounding. What would be the forward price of your contract in Euros? Explain how you would construct such a "quanto" forward contract using the regular (i.e., dollar denominated) forward contract and an appropriate currency forward.

Problem 2 (Put-call parity). 25 points.

Recall gap options, with trigger X and strike K: gap call pays off $S(T)-K$ at maturity, if $S(T)>X$. Analogously, gap put pays off $K-S(T)$ at maturity if $S(T)<X$.

Derive put-call parity (valid at time $t=0$ and at any time t between 0 and maturity T) for gap puts and calls when the strikes K and the triggers X are the same for gap call and put. Do this for:

a) non-dividend paying stock as the underlying and interest rate equal to zero
b) non-dividend paying stock as the underlying and interest rate not equal to zero
c) for stock paying dividends at the rate of $q\%$ per year and interest rate not equal to zero.

With your findings, what is so peculiar about gap options put-call parity? *Hint: use the same portfolio argument as when proving regular put-call parity.*

Problem 3 (Option valuation, BS). 30 points.

Assume that the price of non-dividend paying stock *S(t)* follows GBM, so we have the usual Black-Scholes model. A financial institution offers a new derivative that pays a monetary amount equal to $S(T)^2$ at maturity *T*.

a) Use risk-neutral valuation to calculate the price of this security at time zero and any time *t* between *0* and *T*.

b) In a), there was no optionality in the payoff. Now consider a call option-like payoff
$$\pi(T) = \max(S(T)^2 - K, 0).$$
How this payoff can be valued at time *t=0?*

Problem 4 (Option packages). 20 points.

Consider the following option trading strategy (a package):

Buy a call, strike price A
Sell two calls, strike price B
Buy a call, strike price D

This package is usually bought when underlying is around A.

Draw the payoff diagram of this package. Can this strategy be established at zero cost or at net credit? What is your expectation regarding the underlying price and the implied volatility if you purchase such a package? What is your maximum loss? How does this strategy compare to the butterfly strategy?

Problem 5 (IRS valuation). 15 points.

A financial institution A entered into an interest rate swap with a financial institution B. Under the terms of the swap, A receives 3% per annum and pays 6-month US LIBOR on a principal of 10M USD for 5 years. Payments are semi-annual. Suppose that financial institution B defaults just before the sixth payment date (so at the end of year 3). At that moment, the US LIBOR curve is flat and is 2% p/a for all maturities. The 6-month US LIBOR on the last payment date (so at 2.5 years) was 2.5%. Make no distinction between the discount rate and the reference rate.

What is the loss for financial institution A on this swap due to default of B?

Problem 6 (Alternative models). 30 points.

A popular variant of a mean-reverting process – the so-called Cox-Ingersoll-Ross (CIR) process – is the following: $dX_t = k(\mu - X_t)dt + \sigma\sqrt{X_t}dW_t$.

a) What is the key difference between the regular mean reversion (OU process) and the CIR process? The answer "the square root in front of the Brownian Motion" is not sufficient. Elaborate on the impact of this, i.e., what is the single most important consequence of this extra term for the behavior of this model?

b) It is known that, for the ***regular mean reverting process*** (OU process), the value of the process at time t has a normal distribution with mean $E_0(X_t) = \mu + e^{kt}(X_0 - \mu)$. What is $E_0(X_t)$ in the CIR-model? *Hint: use the integral representation of mean reversion and of CIR process and known result above for MR model.*

c) Suppose that the CIR process describes the evolution of the exponential process $X_t = e^{Y_t}$ for some stochastic process Y_t. What is the stochastic differential equation that Y_t satisfies? Is Y_t also a (form of) mean reversion?