Solutions Exam "Stochastic Processes for Finance" (Dec. 20 2013)

Below, answers (with no or only very brief explanation) to the exam problems are given.

1. (a) The price V_j of the derivative security at time j is, for $0 \leq j \leq N-1$,

$$\frac{1}{2}K\left(\frac{4}{5}\right)^{N-j}.$$

(Use that $\tilde{p} = 1/2$ and hence (under $\tilde{\mathbb{P}}$) the (conditional) probability of an even number of H's is 1/2).

(b) $\Delta_j = 0$ for $0 \leq j \leq N-2$. (This is a consequence of the fact that (see (a)), for $0 \leq j \leq N-1$, V_j does not depend on the outcomes of the coin-flips).

2.

$$f(t,x) = (1 + x + T^2 - t^2)^2 + \frac{T^5}{5} - \frac{t^5}{5}.$$

3. (a) Only for $a = 3$. (That is the only case for which 'the dt term in the differential vanishes').

(b) It is not a martingale. (Use the Ito-Doeblin formula to show that 'the dt term in the differential is non-vanishing').

4. The price at time 0 is

$$2Ke^{-\frac{T}{2}}\left(N\left(\frac{\log 4}{\sqrt{T}}\right) - N\left(\frac{\log 2}{\sqrt{T}}\right)\right).$$

(Use that, by the special choice of σ and r, the 'drift term' for the Brownian motion disappears, so that the problem reduces to considering the distribution of the maximum of a standard Brownian motion over the interval $[0, T]$. Then use the 'reflection principle formula').

5. The price at time 0 is $c(K) - c(2K)$.
(This follows from the fact that $\Phi(y) = (y-K)^+ - (y-2K)^+$).