Department of Mathematics	Exam "Stochastic Processes for Finance"
Vrije Universiteit	December 19, 2008

Give your answers in English.
It is not allowed (nor useful) to use calculators.
Good luck!

1. (Arbitrage arguments)

   Consider a floating rate bond with principal value 1, maturity T, and intermediate payments $C_1, C_2, \ldots, C_n$ at times $T_1 < T_2 < \ldots < T_n = T$ with

   $$C_i = \frac{1}{P(T_{i-1}, T_i)} - 1,$$

   where $P(T_{i-1}, T_i)$ is the price at time T_{i-1} of a T_i-bond. Show that the price at time zero of the floating rate bond is 1.

2. (Discrete time martingales)

   Let $(X_n, n \in \mathbb{Z})$ be a discrete time martingale. Assume furthermore that X is predictable and $X_0 = x_0$. What is the value of X_5 and why?

3. (Brownian motion)

   (a) Let $X_t = cW_{t/c^2}$, where $(W_t, t \geq 0)$ is a Brownian motion and c is a real number. What is the distribution of X_t? Is the process $(X_t, t \geq 0)$ a Brownian motion? Explain why/why not.

   (b) Let $Y_t = \sqrt{t}Z$, where Z is a standard Normal random variable. What is the distribution of Y_t? Is the process $(Y_t, t \geq 0)$ a Brownian motion? Explain why/why not.

   (Hint: check the properties in the definition of Brownian motion.)

4. (Ito's formula)

   Let W denote standard Brownian motion. Show that $X_t = W_t^2 - t$ and $Y_t = W_t^3 - 3tW_t$ are martingales.

5. (Stochastic integration)

   Compute the following stochastic integrals:

   (a) $\int_0^t s\,dW_s$,

   (b) $\int_0^t W_s\,dW_s$,

(c) $\int_0^t W_s^2 \, dW_s$.

(Your answers can be expressed in term of the integral $\int_0^t W_s \, ds$, which you don't need to compute.)

6. (Stock market model)

   Consider a market in which a stock is traded with price process $S_t = e^{\sqrt{2r} W_t}$, where W is a standard Brownian motion under the "real-world" probability measure $\mathbb{P}$ and $r > 0$ is the risk-free interest rate (i.e., $B_t = e^{rt}$).

   (a) Uso Ito's formula to show that the discounted price process $\tilde{S}_t = B_t^{-1} S_t$ is a $\mathbb{P}$-martingale.

   (b) Using (a), give an integral expression for the price V_0 of a derivative with claim $C = f(S_T)$.

7. (Value at Risk)

   Let V_t denote the value of a portfolio at time t and assume that, given the information up to time t, the return $V_{t+\delta t}/V_t - 1$ is normally distributed with mean μ_t and variance σ_t^2. Derive a formula for the Value at Risk (VaR) for the period $[t, t + \delta t]$ with confidence interval $1 - \alpha$.

   (Hint: the answer should be expressed in terms of the inverse function $\Phi^{-1}(1 - \alpha)$, where
   $$\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} \, dx.)$$

Points:

1:	3	2:	3	3(a):	3	4:	4	5(a):	3	6(a):	3	7(a):	3
				3(b):	4			5(b):	3	6(b):	4		
								5(c):	3				