FORMULA SHEET

Erlang distribution Let S_n follow an Erlang(n, μ) distribution. The tail probability of S_n is then

$$\mathbb{P}(S_n > t) = \sum_{k=0}^{n-1} e^{-\mu t} \frac{(\mu t)^k}{k!}.$$

M/M/c queue The probability of waiting Π_W, expectation and distribution of the waiting time W^q and distribution of the sojourn time S

$$\begin{aligned}
\Pi_W &= \frac{(c\rho)^c/c!}{(1-\rho)\sum_{i=0}^{c-1}(c\rho)^i/i! + (c\rho)^c/c!} \\
\mathbb{E}(W^q) &= \Pi_W \frac{1}{c\mu(1-\rho)} \\
\mathbb{P}(W^q > t) &= \Pi_W e^{-c\mu(1-\rho)t} \\
\mathbb{P}(S > t) &= \frac{\Pi_W}{1-c(1-\rho)} e^{-c\mu(1-\rho)t} + \left(1 - \frac{\Pi_W}{1-c(1-\rho)}\right) e^{-\mu t}
\end{aligned}$$

M/M/c/c queue Blocking probability $B(c, a)$, with $a = \lambda/\mu = c\rho$, and relation between Erlang-B and Erlang-C:

$$B(c,a) = \frac{a^c/c!}{\sum_{i=0}^{c} a^i/i!} \qquad \text{and} \qquad \Pi_W = \frac{B(c, c\rho)}{1-\rho+\rho B(c, c\rho)}$$

M/G/1 queue Expected waiting time W^q for FCFS (Pollaczek-Khintchine)

$$\mathbb{E}(W^q) = \frac{\rho}{1-\rho}\frac{\mathbb{E}(B^2)}{2\mathbb{E}(B)} = \frac{1}{2}(1+c_B^2)\mathbb{E}(B)\frac{\rho}{1-\rho}$$

Expected busy period

$$\mathbb{E}(BP) = \frac{\mathbb{E}(B)}{1-\rho}$$

G/M/1 queue Distribution number of customers found upon arrival π^* and expected waiting time W^q

$$\pi_j^* = (1-\sigma)\sigma^j \qquad \text{and} \qquad \mathbb{E}(W^q) = \frac{\sigma}{\mu(1-\sigma)}$$

with σ unique solution in $(0, 1)$ of $\sigma = \mathbb{E}\left[e^{-\mu(1-\sigma)A}\right]$ with A interarrival time

Residual life time Let X be the interarrival time and R be the residual life time. Distribution and expectation of the residual life time R

$$\mathbb{P}(R \leq x) = \frac{1}{\mathbb{E}(X)}\int_0^x \mathbb{P}(X > y)\mathrm{d}y \qquad \text{and} \qquad \mathbb{E}(R) = \frac{\mathbb{E}(X^2)}{2\mathbb{E}(X)}$$