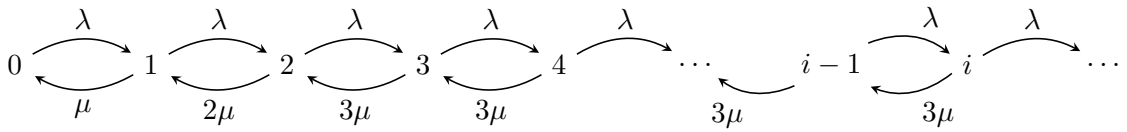
YOUR NAME:
YOUR TA usually:

SOLUTIONS Stochastic Modelling, Short test 4

5 December 2022, 12:25-12:45

Question 1. Consider the $M/M/3$ model. It is a CTMC with transition diagram



(a) Write down the balance equations for sets $\{0, \dots, i-1\}$, $i \geq 1$.

Solution

$$p_0 * \lambda = p_1 * \mu,$$
$$p_1 * \lambda = p_2 * 2\mu,$$
$$\text{for } i \geq 3, \quad p_{i-1} * \lambda = p_i * 3\mu.$$

(b) From the balance equations, express in terms of p_0^{occ} all other p_i^{occ}. You can use the notation $\rho := \lambda/(3\mu)$ (in particular, $\lambda/\mu = 3\rho$, $\lambda/(2\mu) = 3\rho/2$).

Solution

$$p_1 = 3\rho\, p_0,$$
$$p_2 = \frac{3\rho}{2} p_1 = \frac{9\rho^2}{2} p_0,$$
$$\text{for } i \geq 3, \quad p_i = \rho p_{i-1} = \rho^2 p_{i-2} = \ldots = \rho^{i-2} p_2 = \frac{9}{2}\rho^i p_0.$$

Note that the middle formula is compatible with the bottom formula, i.e. we have

$$p_1 = 3\rho\, p_0,$$
$$\text{for } i \geq 2, \quad p_i = \frac{9}{2}\rho^i p_0.$$

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Question 2. In the PK formula, you have to calculate the second moment $E(B^2)$ for the service time B.

(a) What is $E(B^2)$ if the service time B has the normal $N(3, 1^2)$ distribution? Recall that the 1st parameter of the normal distribution is its expectation and the 2nd parameter is its variance.

Solution $\mathbb{E}(B^2) = \mathbb{V}(B) + (\mathbb{E}B)^2 = 1 + 3^2 = 10.$

(b) What is $E(B^2)$ if the distribution of the service time is

$$B \sim \begin{cases} N(3, 1^2) & \text{with probability } 1/3, \\ N(7, 2^2) & \text{with probability } 2/3 \ ? \end{cases}$$

(Show how you plug in all the numbers, you do not have to finish the calculation.)

Solution

$$\mathbb{E}(B^2) = \frac{1}{3}\mathbb{E}(N(3, 1^2))^2 + \frac{2}{3}\mathbb{E}(N(7, 2^2))^2 = \frac{1}{3}10 + \frac{2}{3}(2^2 + 7^2).$$