YOUR NAME:
YOUR TA usually:

Stochastic Modelling, Short test 2

10 October 2022, 12:25-12:45

Question 1. Two types of customers arrive at a service desk. Customers of type A arrive according to a Poisson process of rate λ_A per hour. Customers of type B arrive according to a Poisson process of rate λ_B per hour.

(a) What is the distribution of the inter-arrival times of type A customers? Of type B customers?

In (b) and (c), denote by A_1, A_2, ..., the inter-arrival times of type A customers, and by B_1, B_2, ..., the inter-arrival times of type B customers. Assume that any of the A's is independent from any of the B's.

(b) What is the probability that the first arriving customer is of type A? Explain in terms of "... wins from ...".

(c) What is the probability that the first two customers to arrive are both of type A? Explain in terms "... wins from ...", point it out when you mean a remaining exponential.

In (d) and (e), consider type A customers only. Recall that they arrive according to a Poisson process of rate λ_A per hour.

(d) What is the probability that throughout the first two opening hours (7:00,9:00], exactly one customer of type A arrives?

(e) Are the number of type A customers that arrive during (7:00,9:00] and the number of type A customers that arrive during (8:00,10:00] independent and why?

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Question 2 A production unit consists of 3 machines. Each machine breaks and goes into repair from time to time. Whichever machines are not broken on a given day all work in parallel. There is revenue r generated by a working machine per day, and there are repair costs c per broken machine per day.

Assume that the sequence

$$X_n = \text{number of } \textbf{working} \text{ machines on day } n$$

is a DTMC (with possible state 0, 1, 2, 3) that has both $\pi^{lim} = \pi^{occ} = (\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10})$.

(a) What is the long-run average revenue generated by this system per day? (A formula/expression suffices as the answer, you do not have to simplify it.)

(b) What are the long-run average repair costs of this system per day? (A formula/expression suffices as the answer, you do not have to simplify it.)

(c) Is π^{lim} or π^{occ} relevant in (a) and (b)?