YOUR NAME:
YOUR TA usually:

SOLUTIONS Stochastic Modelling, Short test 2

10 October 2022, 12:25-12:45

Question 1. Two types of customers arrive at a service desk. Customers of type A arrive according to a Poisson process of rate λ_A per hour. Customers of type B arrive according to a Poisson process of rate λ_B per hour.

(a) What is the distribution of the inter-arrival times of type A customers? Of type B customers?

Solution Exponential(λ_A). Exponential(λ_B).

In (b) and (c), denote by A_1, A_2, ..., the inter-arrival times of type A customers, and by B_1, B_2, ..., the inter-arrival times of type B customers. Assume that any A_i is independent from any B_j.

(b) What is the probability that the first arriving customer is of type A? Explain in terms of "... wins from ...".

Solution $P(A_1 \text{ wins from } B_1) = \dfrac{\lambda_A}{\lambda_A + \lambda_B}$

(c) What is the probability that the first two customers to arrive are both of type A? Explain in terms of "... wins from ...", point it out when you mean a remaining exponential.

Solution $P(A_1 \text{ wins from } B_1, A_2 \text{ wins from remaining } B_1) = \dfrac{\lambda_A}{\lambda_A + \lambda_B} \cdot \dfrac{\lambda_A}{\lambda_A + \lambda_B}$

In (d) and (e), consider type A customers only. Recall that they arrive according to a Poisson process of rate λ_A per hour.

(d) What is the probability that throughout the first two opening hours (7:00,9:00], exactly one customer of type A arrives?

Solution $e^{-2\lambda_A} \cdot 2\lambda_A$

(e) Are the number of type A customers that arrive during (7:00,9:00] and the number of type A customers that arrive during (8:00,10:00] independent and why?

Solution No, because these intervals do overlap.

TURN THE PAGE

Question 2 A production unit consists of 3 machines. Each machine breaks and goes into repair from time to time. Whichever machines are not broken on a given day all work in parallel. There is revenue r generated by a working machine per day, and there are repair costs c per broken machine per day.

Assume that the sequence

$$X_n = \text{number of } \textbf{working} \text{ machines on day } n$$

is a DTMC (with possible state 0, 1, 2, 3) that has both $\pi^{lim} = \pi^{occ} = (\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10})$.

(a) What is the long-run average revenue generated by this system per day? (A formula/expression suffices as the answer, you do not have to simplify it.)

Solution $\pi_1^{occ} \cdot r + \pi_2^{occ} \cdot 2r + \pi_3^{occ} \cdot 3r = \dfrac{2}{10} \cdot r + \dfrac{3}{10} \cdot 2r + \dfrac{4}{10} \cdot 3r$

(b) What are the long-run average repair costs of this system per day? (A formula/expression suffices as the answer, you do not have to simplify it.)

Solution $\pi_0^{occ} \cdot 3c + \pi_1^{occ} \cdot 2c + \pi_2^{occ} \cdot c = \dfrac{1}{10} \cdot 3c + \dfrac{2}{10} \cdot 2c + \dfrac{3}{10} \cdot c$

(c) Is π^{lim} or π^{occ} relevant in (a) and (b)?

Solution π^{occ}