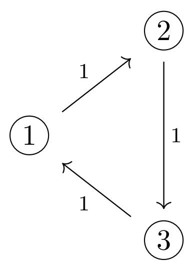
YOUR NAME:
YOUR TA usually:

SOLUTIONS Stochastic Modelling, Short test 1

26 September 2022, 12:25-12:45

Question 1. **(a)** Having started at state 1, does the following DTMC have π^{occ} and π^{lim}? If yes, provide the distribution (an intuitive guess without calculation / motivation is enough); if not, explain why not.



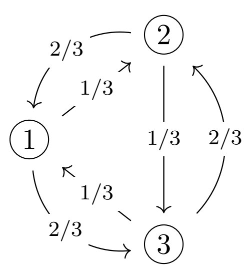
Solution π^{occ} exists, $\pi^{occ} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

π^{lim} does not exist due to periodicity. In more detail, the sequence of transient distributions

$$\pi^{(0)} = (1, 0, 0) \quad *$$
$$\pi^{(1)} = (0, 1, 0) \quad *$$
$$\pi^{(2)} = (0, 0, 1) \quad *$$
$$\pi^{(3)} = (1, 0, 0)... \quad \text{pattern * repeats...}$$

does not have a limit as $n \to \infty$.

(b) For any initial distribution, the following DTMC has both π^{occ} and π^{lim}. Give an intuitive guess for what is the value of $\pi^{occ} = \pi^{lim}$, no calculation / motivation is required.



Solution $\pi^{occ} = \pi^{lim} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ (by symmetry)

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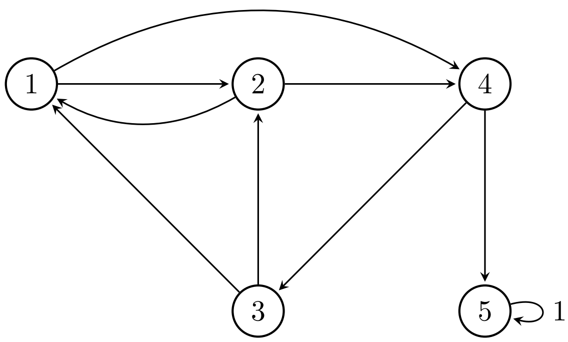
Question 2 Consider a DTMC with the following transition diagram, where all transitions out of states 1, 2, 3, 4 have probability 1/2.



(a) Write down the system of equations that will let you find the expected time to reach (for the 1st time) state 5 from state 1.

Which of the unknowns specifically do you want to know? You do not have to solve the system.

Solution We want to know m_1 from the following system

$$\begin{aligned} m_1 &= 1 + 1/2m_2 + 1/2m_4, \\ m_2 &= 1 + 1/2m_1 + 1/2m_4, \\ m_4 &= 1 + 1/2m_3, \\ m_3 &= 1 + 1/2m_1 + 1/2m_2. \end{aligned}$$

(b) What is the probability that, having started at state 1, the MC will reach state 5 for the 1st time in *exactly* 3 steps? In *at most* 10 steps? If you need a matrix raised to a power in your answer, you can keep the power as is in the answer but make sure to specify the matrix itself.

Solution Let T be the time to reach 5 for the 1st time.

$$\begin{aligned} &P(T = 3|X_0 = 1) \quad \text{the only possible trajectory is } 1 \to 2 \to 4 \to 5 \\ &= p_{12}p_{24}p_{45} = (1/2)^3 = 1/8 \end{aligned}$$

Since state 5 is absorbing,

$$P(T \le 10|X_0 = 1) = (P^{10})_{15},$$

where P is the transition matrix

$$P = \begin{pmatrix} 0 & 1/2 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$