Resit Stochastic Modeling (400646) - Answers

The solutions are always provisionary

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Exercise 1.

(a) Transition diagram [0.5 p]; classes $\{1,2\}$ and $\{3,4\}$ [0.5 p];
$\mathbb{P}(X_2 = 1|X_0 = 1) = \frac{1}{3}$ [1 p] [half p per term]

(b) Yes, both exist [0.5 p]; note that $\pi_1 = \pi_2 = 0$ [0.5 p]
Balance equations [1 p]
Normalization gives result $\pi = (0, 0, 4/7, 3/7)$ [1 p]

(c) Equations [1 p]; final result is 4.5 (solving equations) [1 p]

Exercise 2.

(a) State description X_n = stock level end week n [0.5 p]
State space $\{0, 1, 2, 3\}$, correct P [1.5 p, -0.5 p per incorrect row]

$$\begin{pmatrix} 0 & 1/6 & 1/2 & 1/3 \\ 2/3 & 1/3 & 0 & 0 \\ 1/6 & 1/2 & 1/3 & 0 \\ 0 & 1/6 & 1/2 & 1/3 \end{pmatrix}$$

Alternative: For X_n = stock level start week n (just after receiving order), then state space $\{1, 2, 3\}$ and P:

$$\begin{pmatrix} 1/3 & 0 & 2/3 \\ 1/2 & 1/3 & 1/6 \\ 1/6 & 1/2 & 1/3 \end{pmatrix}$$

(b) The four balance equations [1.5 p]; Prob. lost sales is $\pi_1 1/6$ [0.5 p]

(c) Extend state space by taking into account to order or not, e.g. $Y_n = 1$ (0) order (not) end week n. $\{(X_n, Y_n), n = 0, 1, \ldots\}$ DTMC on $\{0, 1, 2, 3\} \times \{0, 1\}$. [1 p]
Correct P [2 p]

Exercise 3.

(a) $\mathbb{P}(N(0, 5) = 0) = e^{-5\lambda}$ [1 p]
$\mathbb{P}(N_A(0, 5) \geq 2; N_B(0, 5) = 0) = e^{-5/3\lambda}(1 - e^{-10/3\lambda}(1 + \frac{10}{3}\lambda))$ [1 p]

(b)

$$\left(\frac{2/3\lambda}{2/3\lambda + 1/3\lambda}\right)^3 \left(\frac{1/3\lambda}{2/3\lambda + 1/3\lambda}\right) = \frac{8}{81}$$

[2 p]; not working out is -0.5 p

(c) Correct loads of $1/3\lambda$ [0.5 p]; formula's [1 p]

$$\mathbb{E}W_A^q = \frac{1/3\lambda}{2(1 - 1/3\lambda)} = \frac{1}{2}\mathbb{E}W_B^q$$

comparing [0.5 p]; explanation: same load, but A on faster time scale [1 p]

Exercise 4.

(a) Transition diagram [1 p]; giving $B(3, \lambda/\mu)$ [1 p]
Little: $\mathbb{E}L = \lambda(1 - B(3, \lambda/\mu)) \times \frac{1}{\mu}$ [1 p] (for $\mathbb{E}L = \lambda\mathbb{E}S$ 0.5 p)

(b) Transition diagram [0.5 p]; balance equations and $p_i = (\lambda/\mu)^i p_0$ [1.5 p]
Finding p_0 [1 p], writing out all terms is ok

(c)

$$\begin{aligned} \mathbb{P}(W^q > t) &= \pi_1 e^{-\mu t} + \pi_2 \mathbb{P}(S_2 > t) \quad [1p] \\ &= p_0 e^{-\mu_t} \left(\frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \left(\frac{\lambda}{\mu}\right)^2 \mu t\right) \quad [1p] \end{aligned}$$

Exercise 5.

(a) Show $\mathbb{E}B$ [0.5 p]; Derive $\mathbb{E}B^2$ [1 p], show c_B^2 [0.5 p]
Derive $\mathbb{E}W^q = 2 - p$ [1 p]

(b) Sketch [0.5 p]; $\mathbb{E}W^q$ decreasing in p because of less variability [1 p]
$\mathbb{E}L = \frac{1}{4}(4 - p)$ [0.5 p]

(c) Arrival relation [1 p] (in general terms or for parameters)

$$\mathbb{E}W^q = \rho(\mathbb{E}R + \lambda\mathbb{E}R \times \mathbb{E}BP) + (1 - \rho)(\frac{1}{\eta} + \lambda\frac{1}{\eta} \times \mathbb{E}BP)$$

Derive $\mathbb{E}W^q = 2 - p + \frac{1}{\eta}$ [1 p]