Midterm Exam Statistical Methods for CS (X₋401020) and AI (XB₋0080)

Vrije Universiteit Amsterdam, Faculty of Science 18.45 – 20.30h, November 21, 2022

- In general: always motivate your answers.
- Write your answers in English.
- You are allowed to use only a simple, non-graphical, non-programmable calculator.
- The exam is closed-book. Some formulas and tables are available on the last 2 pages of this exam.
- The total number of points available is 40.5 and your midterm grade is the number between 1 and 10 determined as Grade = $1 + \frac{\text{points}}{4.5}$.
- The points are awarded for each question and its parts as follows:

Question	1	2	3	4	5
Part a)	2	2	3	2	3
Part b)	2	1.5	3.5	4	2.5
Part c)	2	3.5	2.5	2	3
Part d)	2	_	_	_	_
Total	8	7	9	8	8.5

- 1. Explain why the following statements are wrong, and reformulate them in a way, such that the statement about convenience sampling, probabilities of unions of events, right-skewed samples, and independence is correct.
 - a) A convenience sample is generally representative for the whole population one is interested in.
 - b) The probability of the union of the events A and B, i.e. $A \cup B$, is equal to the sum of the probabilities P(A) + P(B).
 - c) For right-skewed samples, there are typically many observations much smaller than the sample median, rather than observations which are much greater than the sample median.
 - d) Two independent events are always disjoint but two disjoint events are not necessarily independent.
- 2. A game similar to the game *Wordle* asks the player to guess a 5-letter word. In each round of the game, it is displayed how many and which of the just-typed letters are at the correct position (highlighted in dark grey: A) and which letters appear in the word but are at a wrong position (highlighted in light grey: A).

In the following, we assume for simplicity that the 26 'regular' letters A-Z are used and that also non-existent 5-letter words (like 'AAAAA') can be solution words. All letters (at each position) are assumed equally likely.

a) Give the sample space Ω and the probability measure P for the first round of the game. Hint: because there are too many possibilities, think of an economically feasible way to summarize the sample space. b) Suppose that, after 4 rounds of the game, the following intermediate result has been established: G R A T E

(The first two letters, GR are highlighted in dark grey and the last three letters, ATE, are highlighted in light gray.)

Find the conditional probability that the word will be correctly guessed in the next round, given the available information.

Remember: also non-existent words are allowed.

Also, it is expected of you that all available information is used, e.g. that the first two letters have already been found.

The game uses a scoring system which gives more points if the solution word is guessed in early rounds of the game. The following table summarizes the scores depending on the round of the game and also the (rounded) probabilities (determined from vast records) that the solution word was guessed in a certain round (up to 6). If the word has not been guessed after round 6, the game declares that the word has not been guessed at all and the game ends.

Round k of the game	1	2	3	4	5	6	not at all
Score for guessing the word in round k	1,000	500	200	50	10	1	0
Probability of guessing the word in round k	0	0.01	0.05	0.1	0.2	0.4	0.24

Let the random variable X model the score of the game.

c) Compute the expectation E(X) of X.

Note: for c), state the formula for the expected value that you are using.

3. Suppose that currently 1% of Dutch citizens is acutely infected with SARS-CoV-2. Furthermore, suppose there is an antigen test which correctly detects an actual infection (result: "positive") with a probability of 95% (sensitivity). If one does actually not have the virus, the test correctly confirms this (result: "negative") with a probability of 98% (specificity).

Suppose we ask a randomly selected person to apply the antigen test to him-/herself.

- a) Compute the probability that the test result is positive.
- b) If the test result was positive, compute the conditional probability that this person is indeed infected.
- c) Calculate the probability that, for two *independent* tests applied to the same *actually infected* person, one of the tests is negative and the other is positive.

Note: always name the formulas or theorems that you are using, if they have a name, and repeat the formulas in the way you are using them.

- 4. The computer science student Jake is a board game fan. He would like to calculate or approximate some probabilities and other important parameters that are relevant for his favorite board game. Rolling 6's on a fair 6-sided die plays a major role in this: in particular, each roll of a 6 gives a score of 1, whereas all other die roll outcomes give a score of 0.
 - a) After 100 independent rolls of the fair die, indicate and motivate what is approximately the average score obtained per die roll.
 - b) After 100 independent rolls of the fair die, calculate the (approximate) probability that the average score is less than 0.1.

Tip 1: you may use without further verification that the random variable

$$X_i = score obtained by the i-th die roll, i = 1, ..., 100,$$

has a variance of $\sigma^2 = 5/36$.

Tip 2: the result from a) also plays a role in finding the solution in b). If you were not able to solve a), you may (wrongly) assume for solving b) that the answer in a) is 0.2.

c) Unrelated to the previous questions: find the probability that a standard normally distributed random variable is bigger than 1.24.

Note: always name the formulas or theorems that you are using, if they have a name, and repeat the formulas in the way you are using them.

5. a) The two plots in Figure 1 below display two density functions each. Make qualitative comparisons of densities 1 and 2 and also of densities 3 and 4 by pointing out the most important similarities and differences.

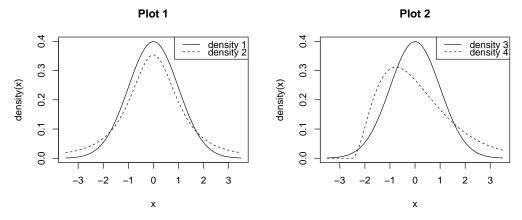


Figure 1: In each plot, two model distributions are compared.

- b) Draw a boxplot that represents a random sample (of sufficiently large size) drawn from the distribution illustrated by density4. Note: it is impossible to draw a completely accurate boxplot; but the most important features should be visible.
- c) What is a QQ-plot: Describe what's on the axes, what you can check with a QQ-plot, and how you can check that. In Figure 2 we see a *normal QQ*-plot. What can you conclude about the tails of the data distribution shown in this plot.

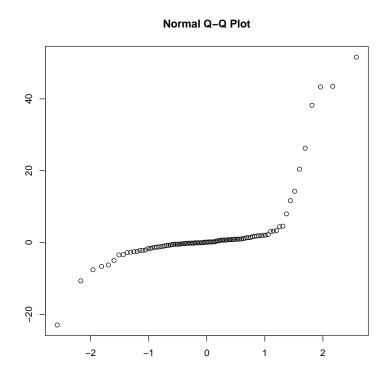


Figure 2: Normal QQ-plot. The labels of the axes are intentionally left blank, because that's question 2c.

Formulas and Tables for Exam Statistical Methods

Probability

We use the following notation:

 Ω sample space, P probability measure.

 B, A_1, A_2, \ldots, A_m events,

 A_1, A_2, \ldots, A_m a partition of Ω with $P(A_i) > 0$ for all $i \in \{1, 2, \ldots, m\}$.

Law of Total Probability:

$$P(B) = \sum_{i=1}^{m} P(B \cap A_i) = \sum_{i=1}^{m} P(B|A_i)P(A_i).$$

Bayes' Theorem:

$$P(A_r|B) = \frac{P(A_r \cap B)}{\sum_{i=1}^m P(B|A_i)P(A_i)} = \frac{P(B|A_r)P(A_r)}{\sum_{i=1}^m P(B|A_i)P(A_i)}.$$

Z

Decide	Table 2	Standard No	/-/	SUIDAHOII.	umulative A	Standard Normal (z) Distribution: Cumulative Area from the LEFT	LEF!				
1,0003	1	00.	10.	.02	.03	.04	.05	90.	70.	80.	60:
5 .0003 .0003 .0003 .0003 .0003 .0003 .0003 .0003 .0004 .0	50 and	.0001	E S	8	100	183	Darr	paid . pag	105	+0+	
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5 .0006 .0006 .0006 .0006 .0006 6 .0009 .0008 .0008 .0006 .0006 8 .0017 .0012 .0011 .0011 .0011 .0011 8 .0017 .0016 .0016 .0016 .0019 .0007 9 .0017 .0016 .0016 .0011 .0011 .0010 1 .0023 .0023 .0022 .0021 .0015 .0014 1 .0043 .0042 .0029 .0029 .0029 .0021 2 .0043 .0042 .0021 .0021 .0021 .0021 3 .0055 .0044 .0029 .0028 .0028 .0027 4 .0057 .0049 .0049 .0049 .0049 .0049 .0049 5 .0126 .0052 .0052 .0059 .0058 .0069 6 .0127 .0077 .0027 .0049 <t< td=""><td>-3.3</td><td>9000.</td><td>.0005</td><td>3000.</td><td>.0004</td><td>.0004</td><td>.0004</td><td>.0004</td><td>.0004</td><td>.0004</td><td>.0003</td></t<>	-3.3	9000.	.0005	3000.	.0004	.0004	.0004	.0004	.0004	.0004	.0003
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1515	F	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
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2327 2296 2286 2276 2177 6 2843 2841 2844 2448 2 3386 3390 3264 2877 2843 2810 2 34880 4405 4404 4484 4486 4486 4486 4486	6.0	1841.	1814	8871.	29/1.	1/36	1171.	6801.	1000	1894	186
6 2643 2611 2578 2546 2514 2483 5 2981 2946 2912 2877 2843 2810 2 3336 3300 3264 3228 3192 3156 5 3707 3669 3632 3594 3857 3820 9 4090 4052 4013 3974 3836 3897 0 4880 4443 4404 4364 4325 4286 0 4880 4840 4801 4761 4721 4681	0.0	2420	2389	2358	2327	.2296	.2266	.1236	.2206	7712.	.2148
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2 .3336 .3264 .3228 .3192 .3156 5 .3707 .3669 .3632 .3594 .3557 .3520 9 .4090 .4052 .4013 .3974 .3336 .3897 2 .4489 .4443 .4404 .4364 .4325 .4286 0 .4880 .4840 .4801 .4761 .4721 .4681	-0.5	3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
3821 3784 3775 3869 3692 3594 3557 3520 3520 3692	-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
9 4090 4052 4013 3974 3996 3897 2 4483 4404 4364 4255 4286 0 4880 4840 4801 4761 4721 4681	-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
2 .4483 .4404 .4384 .4386 .4286 .7286 . 0 .4880 .4840 .4801 .4761 .4721 .4681	-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
	-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
	0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
2 Soote Atea	rE: For value	es of z below 3.4 nmon values tha	19, use 0.0001 fc t result from inte	or the area.				^			(continued)
	z Score	Area									
	1707										

POSITIVE z Scores

0.0.0. 5,000 5,040 5,050 5,150 5,150 5,150 5,525 5,52			10.	707	.03	.04	.05	90.	.07	80.	BO:
5517 56567 5696 5636 5646 5672 <t< td=""><td></td><td>2000</td><td>.5040</td><td>.5080</td><td>.5120</td><td>.5160</td><td>.5199</td><td>.5239</td><td>.5279</td><td>.5319</td><td>.5359</td></t<>		2000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
5910 5948 5887 6026 6664 6700 6772 606 7019 7054 7708 77123 77019 7704 7724 7744 7702 7704 7724 7744 7703 7704 7773 7744 7704 7724 7744 7744 7704 7724 7744 7744 7704 7724 7744 7744 7704 7724 7744 7774 7708 8256 8907 8254 8051 8007 8256 8944 8962 8279 8008 8952 8944 8962 8279 8008 8971 3948 3946 3946 8009 8971 3948 3946 3946 8001 3960 3960 3946 3946 802 3960 3966 3966 802 3966 3966 3966 <td></td> <td>5398</td> <td>.5438</td> <td>.5478</td> <td>.5517</td> <td>.5557</td> <td>.5596</td> <td>.5636</td> <td>.5675</td> <td>.5714</td> <td>.5753</td>		5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
6.6263 6.8381 6.8066 6.406 6.406 6.406 6.406 6.406 6.772 6.772 6.772 6.772 6.772 6.772 6.772 6.772 6.772 6.772 6.772 7.744 7.7734 7.7744		5793	.5832	.5871	.5910	.5948	7865.	.6026	.6064	.6103	.6141
7.719 7.7054 7.708 7.7123 7.719 7.7054 7.708 7.7123 7.727 7.734 7.744 7.744 8.828 8.8264 8.8289 8.815 8.845 8.8284 8.854 8.854 8.807 8.828 8.844 8.862 8.807 8.828 8.870 8.870 8.807 8.828 8.844 8.862 8.807 8.828 8.844 8.862 8.807 8.828 8.844 8.862 8.807 8.828 8.844 8.862 8.807 8.894 8.862 8.871 8.808 8.828 8.844 8.862 8.807 8.898 8.871 8.862 8.808 8.894 8.862 8.864 8.809 8.894 8.862 8.862 8.809 8.894 8.862 8.862 8.809 8.894 8.862 8.862 8.807		6119	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
7.719 7.7054 7.708 7.123 7.757 7.734 7.754 7.754 8.828 8.8264 8.8289 8.815 8.485 8.8284 8.854 8.854 8.807 8.828 8.815 8.854 8.807 8.828 8.815 8.854 8.807 8.828 8.815 8.854 8.807 8.828 8.815 8.854 8.807 8.828 8.815 8.854 8.807 8.828 8.817 8.870 8.807 8.828 8.844 8.862 8.807 8.828 8.844 8.862 8.808 8.828 8.870 8.968 8.807 8.898 8.944 8.962 8.808 8.944 8.962 9.978 8.944 8.944 8.962 9.978 8.944 8.946 9.948 9.948 8.944 8.946 9.948 9.948 8.944 <		6554	.6591	.6628	.6664	.6700	.6736	.6772	9089	.6844	6289
7.7567 7.7389 7.7424 7.7673 7.7704 7.7734 7.7764 8.828 8.8264 8.8289 8.815 8.485 8.608 8.815 8.870 8.807 8.8289 8.815 8.870 8.807 8.8281 8.854 8.870 8.807 8.8281 8.854 8.870 8.807 8.8281 8.854 8.870 8.807 8.8281 8.854 8.870 8.807 8.8281 8.854 8.870 8.807 8.8281 8.854 8.870 8.808 8.874 8.870 8.871 8.807 8.8281 8.864 8.862 8.808 8.894 8.962 9.278 9.808 9.894 9.896 9.896 9.808 9.894 9.894 9.894 9.808 9.894 9.894 9.894 9.809 9.894 9.894 9.894 9.809 9.894 </td <td></td> <td>6915</td> <td>.6950</td> <td>.6985</td> <td>.7019</td> <td>.7054</td> <td>.7088</td> <td>.7123</td> <td>.7157</td> <td>.7190</td> <td>.7224</td>		6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
2 77673 7774 7774 7774 3 7767 7794 7774 7764 4 8288 8281 8051 8051 8 8455 8528 8315 870 8 8729 8749 8770 8 8907 8925 8749 8770 8 8907 8925 8749 8770 8 8907 8925 8749 8770 9 8907 8926 8934 8962 10 8944 8962 8578 8978 10 8944 8962 8578 8978 10 8944 8962 8978 8978 10 8944 8962 8978 8978 10 8944 8962 8968 8968 10 8944 8962 8968 8968 10 8944 8964 8962 8964 10 89		7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
2 .8284 .8051 2 .8286 .8531 .8554 3 .8485 .8531 .8554 8 .8729 .8749 .8770 8 .8907 .8925 .8749 .8770 8 .9082 .9084 .8962 .8770 8 .9082 .9251 .9265 .9279 8 .9082 .9284 .8962 .9713 9 .9251 .9265 .9279 .9718 1 .9484 .9406 .9608 .9616 2 .9372 .9738 .9744 .9750 3 .978 .978 .9616 .9608 9 .9773 .9978 .9846 .9846 9 .9877 .9986 .9986 .9986 9 .9943 .9984 <t>.9984 .9986 9 .9944 .9948 .9986 .9986 9 .9986 .9986</t>	123	7580	.7611	.7642	.7673	.7704	.7734	7764	7794	.7823	.7852
2 8286 8289 8315 3 8485 8628 8315 8485 8608 8531 8554 8907 8925 8944 8962 9082 3093 3115 3131 2 3926 3934 3962 3927 3938 3934 3906 3943 3949 3949 3940 3943 3948 3948 3948 3944 3949 3946 3946 3942 3949 3948 3948 3943 3948 3948 3948 3944 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 3948 <td< td=""><td></td><td>7881</td><td>.7910</td><td>.7939</td><td>7967.</td><td>7995</td><td>.8023</td><td>.8051</td><td>8078</td><td>.8106</td><td>.8133</td></td<>		7881	.7910	.7939	7967.	7995	.8023	.8051	8078	.8106	.8133
8408 .8531 .8554 8708 .8729 .8749 .8770 8907 .8925 .8944 .8962 9082 .9039 .9115 .9131 1926 .9251 .9265 .979 1926 .9251 .9265 .979 2000 .9751 .9406 .975 2000 .9732 .9738 .9668 2004 .9674 .9678 .9668 2007 .9838 .9744 .9750 2008 .9973 .9942 .9948 2009 .9942 .9948 .9948 2009 .9948 .9948 .9948 2009 .9948 .9948 .9948 2009 .9948 .9948 .9948 2009 .9948 .9948 .9948 2009 .9948 .9984 .9948 2009 .9948 .9989 .9988 2009 .9998 .9989 .	L-1	8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
87708 .8729 .8749 .8770 8907 .8925 .8944 .8962 9082 .9039 .9115 .9131 18970 .9251 .9265 .9279 18970 .9382 .8394 .9406 1892 .9562 .9739 .9406 1892 .9562 .9739 .9608 1893 .9744 .9750 .9608 1894 .9873 .9474 .9666 1894 .9873 .9942 .9968 1894 .9942 .9948 .9948 1898 .9948 .9946 .9946 1898 .9948 .9948 .9948 1898 .9989 .9989 .9989 1898 .9989 .9986 .9986 1898 .9986 .9986 .9986 1898 .9986 .9986 .9986 1898 .9986 .9986 .9986 1898 .9986		8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
8907 .8925 .8944 .8962 9082 .9039 .9115 .9131 7 .925 .925 .925 .9279 8 .9484 .9495 .9406 .975 8 .9664 .9671 .9678 .9406 9 .9672 .9738 .9678 .9668 9 .9732 .9738 .9674 .9678 9 .9732 .9744 .9750 9 .9873 .9744 .9760 9 .9874 .9803 .9846 9 .9974 .9968 .9969 9 .9977 .9978 .9948 9 .9966 .9969 .9969 9 .9988 .9989 .9986 9 .9996 .9996 .9996 9 .9996 .9996 .9996 9 .9996 .9996 .9996 9 .9996 .9996 .9996		8643	.8665	9898	8028.	.8729	.8749	.8770	0628.	.8810	.8830
2 3908 3115 3111 3 3256 3251 3265 3279 4 3444 3436 3606 3615 379 3 3664 3671 3689 3608 3616 3 3672 3738 3744 3750 3 3778 3789 3608 3666 3 3778 3875 3946 3600 3 3875 3942 3946 3946 4 3967 3956 3960 3961 5 3968 3969 3961 3971 6 3967 3967 3976 3971 7 3988 3989 3989 3989 8 3994 3998 3989 3989 9 3998 3988 3989 3989 9 3998 3998 3999 3996 9 3999 3999 3999 3999 <tr< td=""><td></td><td>8849</td><td>.8869</td><td>.8888</td><td>7068.</td><td>.8925</td><td>.8944</td><td>.8962</td><td>.8980</td><td>7668.</td><td>.9015</td></tr<>		8849	.8869	.8888	7068.	.8925	.8944	.8962	.8980	7668.	.9015
2 3926 3926 3926 3979 3 3484 3496 3406 3406 3 3484 3496 3406 3416 3 3622 3534 3406 3416 3 3623 3639 3688 3688 3 3738 3744 3750 3 3778 3678 3686 3 3778 3676 3676 3 3873 3744 3750 3 3873 3744 3750 3 3874 3846 3846 3901 3904 3946 3946 3943 3946 3946 3946 3948 3969 3946 3946 3948 3988 3989 3989 4 3994 3994 3996 5 3996 3996 3996 6 3996 3996 3996 7 3998		9032	.9049	9906	.9082	6606	.9115	.9131	.9147	.9162	.9177
4 .9484 .9496 .9406 3 .9582 .9598 .9406 3 .9582 .9599 .9615 6 .9722 .9738 .9744 .9750 7 .9838 .9744 .9750 8 .9732 .9738 .9846 9 .9873 .9744 .9750 1 .9874 .9878 .9846 1 .9877 .9878 .9846 2 .9877 .9928 .9948 3 .9945 .9948 .9946 3 .9948 .9948 .9949 4 .9948 .9948 .9948 9 .9969 .9989 .9989 1 .9994 .9994 .9994 2 .9986 .9986 .9986 3 .9986 .9986 .9986 4 .9994 .9994 .9994 5 .9996 .9996 .9996 <td></td> <td>9192</td> <td>.9207</td> <td>.9222</td> <td>.9236</td> <td>.9251</td> <td>.9265</td> <td>.9279</td> <td>.9292</td> <td>.9306</td> <td>.9319</td>		9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
4 .9484 .9495 .9505 .9515 .9599 .9618 5 .9664 .9671 .9599 .9608 .9608 6 .9664 .9678 .9678 .9686 .9608 3 .9728 .9738 .9744 .9750 3 .9788 .9673 .9646 .9671 8 .9917 .9875 .9842 .9846 9 .9927 .9928 .9946 .9946 1 .9943 .9946 .9946 .9946 2 .9967 .9969 .9960 .9967 3 .9984 .9984 .9984 .9984 4 .9994 .9994 .9994 .9994 5 .9986 .9996 .9996 6 .9996 .9996 .9996 7 .9987 .9994 .9994 8 .9996 .9996 .9996 9 .9996 .9996 .9996 <td></td> <td>9332</td> <td>.9345</td> <td>.9357</td> <td>.9370</td> <td>.9382</td> <td>.9394</td> <td>.9406</td> <td>.9418</td> <td>.9429</td> <td>.9441</td>		9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
3.9582 .9591 .9589 .9608 69664 .9671 .9678 .9678 .9686 3.9722 .9738 .9744 .9750 3.9873 .9748 .9803 6.9871 .9875 .9842 .9846 8.9871 .9876 .9846 .9861 8.9971 .9947 .9966 .9969 9.9673 .9946 .9946 .9946 .9946 9.9674 .9969 .9960 .9967 .9947 6 .9967 .9969 .9960 .9967 7 .9988 .9984 .9984 .9984 8 .9994 .9994 .9994 .9994 9 .9986 .9986 .9986 1 .9994 .9994 .9994 2 .9986 .9986 .9986 3 .9986 .9986 .9986 4 .9994 .9994 .9994 5 .9986		.9452	.9463	.9474	.9484	* 3495 *	.9505	.9515	.9525	.9535	.9545
6 .9664 .9671 .9678 .9686 3 .9722 .9738 .9744 .9750 1 .9824 .9873 .9798 .9803 1 .9874 .9878 .9866 .9803 2 .9871 .9878 .9861 .9861 2 .9925 .9906 .9906 .9906 2 .9925 .9946 .9948 .9948 3 .9946 .9960 .9961 .9961 4 .9987 .9989 .9989 .9989 5 .9986 .9986 .9989 .9989 6 .9994 .9994 .9994 .9994 7 .9996 .9989 .9989 .9989 8 .9996 .9996 .9996 .9996 9 .9996 .9996 .9996 .9996 9 .9996 .9996 .9996 .9996 9 .9996 .9996 .9996 <td>53.4</td> <td>9554</td> <td>.9564</td> <td>.9573</td> <td>.9582</td> <td>.9591</td> <td>.9599</td> <td>8096</td> <td>.9616</td> <td>.9625</td> <td>.9633</td>	53.4	9554	.9564	.9573	.9582	.9591	.9599	8096	.9616	.9625	.9633
3.9732 .9738 .9744 .9750 3.9738 .9733 .9798 .9603 3.934 .9838 .9845 .9846 8.9971 .9875 .9878 .9846 8.9971 .9904 .9906 .9909 2.9925 .9927 .9948 .9948 3.9943 .9946 .9946 .9948 4.9957 .9959 .9948 .9948 5.9968 .9969 .9997 .9977 6.9996 .9996 .9996 .9996 7.9994 .9994 .9994 .9994 8.9996 .9996 .9996 .9996 9.9996 .9996 .9996 .9996 9.9996 .9996 .9996 .9996 9.9996 .9996 .9996 .9996 9.9996 .9996 .9996 .9996 9.9996 .9996 .9996 .9996 9.9997 .9997 .9997 .9996		.9641	.9649	.9656	.9664	.9671	8296.	9896	.9693	6696	9026.
3 .9788 .9798 .9808 9 .9824 .9838 .9846 .9846 8 .9871 .9875 .9846 .9846 9 .9901 .9904 .9906 .9909 2 .9925 .9927 .9929 .9931 1 .9943 .9946 .9948 .9948 2 .9957 .9959 .9948 .9948 3 .9968 .9969 .9986 .9986 4 .9994 .9994 .9994 .9994 5 .9986 .9986 .9986 .9986 7 .9988 .9984 .9994 .9994 8 .9986 .9986 .9986 .9986 9 .9994 .9994 .9994 .9994 1 .9987 .9994 .9994 .9994 2 .9986 .9996 .9996 .9996		.9713	9719	.9726	.9732	.9738	.9744	.9750	9226	.9761	2926.
88.4 .98.46 .98.46 88.71 .98.75 .98.76 .98.16 89.901 .9904 .9906 .9909 19.943 .9945 .9946 .9948 19.943 .9946 .9948 .9948 29.957 .9959 .9948 .9948 29.968 .9969 .9971 .9971 29.988 .9984 .9984 .9984 39.998 .9986 .9986 .9986 4 .9994 .9994 .9994 .9996 5 .9996 .9996 .9996 .9996 7 .9998 .9994 .9994 .9994 8 .9996 .9996 .9996 9 .9996 .9996 .9996 1 .9994 .9994 .9994		9772	8776.	.9783	.9788	.9793	92.6	.9803	8086	.9812	.9817
8 .9871 .9875 .9878 .9881 9 .9901 .9904 .9906 .9908 1 .9943 .9945 .9946 .9948 1 .9967 .9969 .9960 .9971 2 .9968 .9969 .9977 .9974 3 .9983 .9984 .9986 .9986 4 .9994 .9994 .9994 .9994 5 .9996 .9996 .9996 6 .9996 .9996 .9996 7 .9994 .9994 .9994 8 .9996 .9996 .9996 9 .9997 .9997 .9997		9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2 .9904 .9904 .9906 .9908 2 .9925 .9927 .9945 .9948 1 .9943 .9945 .9946 .9948 2 .9968 .9969 .9971 .9971 2 .9968 .9969 .9949 .9949 3 .9948 .9984 .9986 .9986 4 .9994 .9994 .9994 .9994 5 .9996 .9996 .9996 6 .9996 .9996 .9996 7 .9997 .9994 .9994 8 .9996 .9996 .9996 9 .9997 .9997 .9997		19861	.9864	.9868	.9871	9875	9878	.9881	.9884	.9887	.9890
2 .9926 .9927 1 .9943 .9945 .9946 2 .9967 .9969 .9960 3 .9977 .9977 .9978 2 .9983 .9984 .9984 3 .9986 .9986 .9986 4 .9994 .9994 .9994 5 .9996 .9996 .9996 6 .9996 .9996 .9996 7 .9997 .9996 .9996 8 .9996 .9996 .9996 9 .9997 .9997 .9997		.9893	9686	8686.	1066.	9304	9066	6066	.9911	.9913	.9916
1 .9943 .9945 .9946 .9948 2 .9957 .9959 .9970 .9971 2 .9977 .9978 .9979 .9979 2 .9983 .9984 .9984 .9985 3 .9981 .9982 .9982 .9982 4 .9994 .9994 .9994 .9996 5 .9996 .9996 .9996 .9996 7 .9997 .9997 .9997		9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	9836
7 3967 3959 3960 3961 2 3977 3977 3977 3978 3979 2 3988 3988 3988 3989 1 3994 3994 3994 2 3996 3996 3996 2 3997 3997 3 3997 3997 3 3997 3997	-	.9938	.9940	.9941	.9943	.9945	.9946	.9948	* 6566.	.9951	.9952
7 3968 3970 3971 3977 3978 3979 3979 3979 3988 3988 3988	5.6	.9953	.9955	.9956	7366.	.9959	0966	.9961	.9962	.9963	.9964
2 .9977 .9978 .9988 .9988 2 .9983 .9984 .9985 .9985 7 .9988 .9989 .9989 .9989 1 .9994 .9994 .9994 .9996 2 .9996 .9996 .9996 3 .9997 .9997	2.7	3962	9966	7966.	8966	6966	0266.	.9971	.9972	.9973	9974
2 3983 3984 3984 3985 7 3988 3988 3988 3989 1 3994 3994 3994 3996 5 3996 3996 3996 7 3997 3997		.9974	366.	9266.	7266.	7266.	8266.	6266	6266.	0866	.9981
7 3988 3988 3988 3989 3989 3989 3989 4 3989 3984 39894 39896 39896 39896 39896 39896 39896 39897 39897	2.9	.9981	.9982	.9982	.9983	.9984	.9984	3885	3885	9866	9866
1 3992 3994 3994 3994 3994 3996 3996 3996 3996	3.0	7866.	7866.	7866.	.9988	.9988	6866	6866	6866	0666	0666.
4 .9994 .9994 .9994 .9994 .9994 .9996 .7996 .9996 .7 .9997 .7	3.1	0666	1666.	.9991	1666.	3666.	3666	3885	2666.	.9993	.9993
9866. 9897. 7898. 7898. 7	3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	3666	3666	3885
7898. 7898. 7	3.3	3995	3666	3666.	9666	9666	9666	9666	9666	9666	7666.
3.50 and up 3999 WOTE: For values of z above 3.49, use 0.9999 for the area. Use these common values that result from interpolation: Z Soore Area	3.4	79997	7666.	7666.	2666.	7666.	7666.	7666.	7666.	7666.	8666.
WOTE: For values of zabove 3.49, use 0.9999 for the area. "Use these common values that result from interpolation: Z Score Area		6666			444	10804					
2 Sore Area	NOTE: For values of .	z above 3.49,	use 0.9999 f	or the area.	i	ĺ	· ·			Common Critical Values	itical Valı
	Score	Area				ĺ				Confidence	Critical
1 6.45 D 950D €	1 645	0.9500							D.	06:0	1.645
	2.575	0.9950	,	B	e e	ě				0.95	1.96