Final Exam Statistical Methods

Vrije Universiteit Amsterdam, Faculty of Exact Sciences

15.15 – 18.00h, December 15, 2015

- Always *motivate* your answers.
- Write your answers in English.
- Only the use of a simple, non-graphical calculator is allowed.
- Programmable/graphical calculators, laptops, mobile phones, smart watches, books, own formula sheets, etc. are not allowed.
- On the last four pages of the exam, some formulas and tables that you may want to use can be found.
- The total number of points you can receive is 63: $\text{Grade} = 1 + \frac{\text{points}}{7}$.
- The division of points per question and subparts is as follows:

Question	1	2	3	4	5	6
Part a)	1	2	2	2	3	3
Part b)	2	2	8	8	2	1
Part c)	2	3	2	3	3	7
Part d)	-	-	-	-	3	4
Total	5	7	12	13	11	15

- If you are asked to perform a test, do not only give the conclusion of your test, but report:
  1. the hypotheses in terms of the population parameter of interest;
  2. the significance level;
  3. the test statistic and its distribution under the null hypothesis;
  4. the observed value of the test statistic;
  5. the P-value or the critical value(s);
  6. whether or not the null hypothesis is rejected and why;
  7. finally, phrase your conclusion in terms of the context of the problem.

Good luck!

1. Assume that the body weight in a population is a normally distributed random variable with unknown mean μ and unknown standard deviation σ. Suppose that the body weight of 16 randomly selected people from that population is measured and that the sample mean is $\bar{x} = 73.34$, and the sample standard deviation is $s = 12.00$.

   a) Explain why you should use a t-distribution to construct a confidence interval for μ.

   b) A confidence interval for the unknown population mean μ is given by $[68.081, 78.599]$. Is the confidence level 95% or 90%?

   c) What is the interpretation of this confidence interval, given your answer in part b)?

2. Before the permanent closing of the helpdesk of Education Office VU Science, 100 randomly selected students of the Faculty of Exact Sciences (FEW) were asked whether or not they like this change, and 77 students liked it. Let p be the proportion of FEW-students who like the change.

   a) Determine the usual point estimate $\hat{p}$ of p.

   b) Give the 90% confidence interval based on the results of the poll among the 100 students.

   c) Use the estimate from part a) to determine the minimal number of students that should be surveyed to obtain a 95% confidence interval with the same margin of error as the confidence interval from part b). [If you cannot answer part b), use $E = 0.050$. NB. This is not the correct value.]

3. In a study on the effects of social networking on cognitive skills two groups of 36 Facebook users each were tested. Subjects in the first group were asked to deactivate their accounts for two weeks; subjects in the second group did not alter their usual Facebook activity. After two weeks, both groups were shown a 5 minute video clip and then asked to answer some questions about the clip. The mean number of correctly answered questions was used to measure cognitive skills. For the first group the mean and standard deviation of the number of correct answers were $\bar{x}_1 = 7.98$ and $s_1 = 1.07$; for the second group $\bar{x}_2 = 8.82$ and $s_2 = 2.20$. Some other characteristics of the two samples that you may or may not use are: $\bar{d} = -0.84$, $s_d = 2.30$, $s_p = 1.73$.

   a) Are these matched-pair or independent samples?

   b) Perform a test to investigate the claim that social networking has an effect on cognitive skills. Take significance level $\alpha = 0.1$. (See the first page of this exam for detailed instructions about testing).

   c) The test you performed in part b) should only be used if certain requirements are met. What are these requirements and are they met in this case?

4. During one week in September 2015, The Dutch Cyclists' Union (*De Fietsersbond*) used an app called 'App de Fiets!' to collect various data about biking habits in the Netherlands. For instance, it turned out that the daily average biking distance of the users of the app is 5.56 km. Independent samples were taken of 240 users of the app from the province of South Holland, and 300 from the province of North Holland. In

the South Holland sample, 49 people biked on average more than 5.56 km in one day; in the North Holland sample, 77 people biked on average more than 5.56 km in one day. Let p_1 denote the proportion of app users in South Holland whose daily average is higher than 5.56 km and let p_2 denote the proportion of app users in North Holland whose daily average is higher than 5.56 km.

a) Give based on the data a point estimate for the difference between the proportions of app users in South Holland whose daily average exceeds 5.56 km and of app users in North Holland whose daily average exceeds 5.56 km.

b) Perform a suitable test using the P-value method to investigate the claim that the proportion of app users whose daily average exceeds 5.56 km is higher in North Holland than in South Holland. Take significance level $\alpha = 0.05$. (See the first page of this exam for detailed instructions about testing).

c) What is 'Type I error'? What is 'Type II error'? Is any of the two errors related to the significance level? How?

5. A statistics teacher asked 50 randomly selected students following her statistics courses whether they prefer working in *R* with or without *RStudio*. In addition, she also asked them whether they are Windows users or not. The results she found are presented in the following table:

	Windows	other	total
pure *R*	7	13	20
RStudio	18	12	30
total	25	25	50

a) Does this concern a test for homogeneity or a test for independence of variables? Formulate the null and alternative hypothesis.

b) Compute the expected frequencies table under the assumption that the null hypothesis is true.

c) Use part b) to show that the requirements for a suitable chi-square test are satisfied. Give a formula of the chi-square test statistic and its distribution under H_0.
(*You do not have to calculate the value of the test statistic.*)

d) The observed value of the test statistic is $\chi^2 = 3.00$. What is your conclusion for significance level $\alpha = 0.05$?

6. The new ICON E-Flyer is an electric bike, developed in the US, that can reach a speed of 60 km/h. We wish to investigate how this maximum speed depends on the windspeed in the opposite direction (in Bft). The maximum speed and the windspeed for 25 rides were measured and stored in the respective dataset y and x. A linear regression analysis was carried out with explanatory variable 'windspeed' and response variable 'maximum speed'. Some characteristics of the data that you may or may not use are:

$$\overline{x} = 5.34, \quad \overline{y} = 43.37, \quad s_x = 1.60, \quad s_y = 3.68,$$

$$s_{b_0} = 1.16, \quad s_{b_1} = 0.21, \quad r = -0.90, \quad \sqrt{\frac{1-r^2}{n-2}} = 0.091.$$

Furthermore, a scatterplot of the maximum speed against the windspeed, as well as other plots related to the regression analysis are shown in Figure 1.

a) Based on these data, give an estimate for the regression equation. What is the predicted maximum speed for the windspeed of 5 Bft?

b) How much of the variation in the response variable can be approximately accounted for by the explanatory variable, i.e., by the regression line?

c) Test the claim that the population linear correlation coefficient equals 0. Take significance level $\alpha = 0.01$. (See the first page of this exam for detailed instructions about testing).

d) Suppose that you were to test the claim that $\beta_1 = 0$. For this test certain requirements about the errors have to be met, and two of them can be verified with the middle and right plot in Figure 1. What are these assumptions? Is it reasonable to assume that they are indeed met?

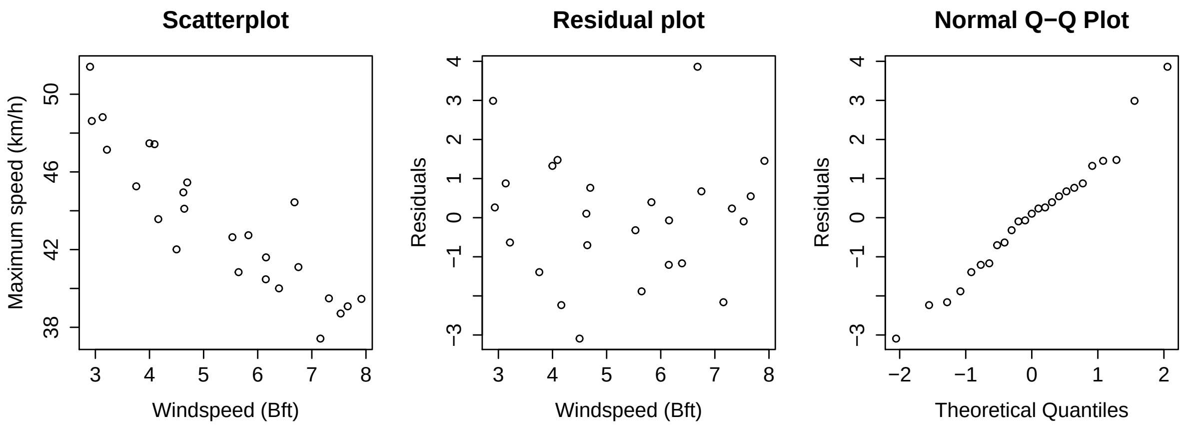


Figure 1: Scatterplot, residual plot, normal QQ plot of residuals.

Formulas and Tables for Exam Statistical Methods

Two *independent* samples

(The statements below hold if certain requirements are met. You should always verify these requirements first.)

For two *independent* samples,

(i) if σ_1 and σ_2 are unknown and $\sigma_1 \neq \sigma_2$, the test statistic

$$T_2 = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}$$

has a t-distribution with approximately $\tilde{n}$ degrees of freedom under the null hypothesis. We use the conservative estimate $\tilde{n} = \min\{n_1 - 1, n_2 - 1\}$.

(ii) if σ_1 and σ_2 are unknown and $\sigma_1 = \sigma_2$, then the test statistic

$$T_2^{\text{eq}} = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2/n_1 + S_p^2/n_2}}$$

has a t-distribution with $n_1 + n_2 - 2$ degrees of freedom under the null hypothesis. Here S_p^2 is the pooled sample variance given by

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}.$$

(iii) if $p_1 = p_2$, the test statistic

$$Z_p = \frac{(\hat{P}_1 - \hat{P}_2) - (p_1 - p_2)}{\sqrt{\overline{P}(1 - \overline{P})/n_1 + \overline{P}(1 - \overline{P})/n_2}}$$

approximately has a standard normal distribution. Here $\overline{P} = (X_1 + X_2)/(n_1 + n_2)$ is the pooled sample proportion.

(iv) the margin of error for a $1 - \alpha$ confidence interval for $p_1 - p_2$ is given by

$$E = z_{\alpha/2}\sqrt{\hat{p}_1(1 - \hat{p}_1)/n_1 + \hat{p}_2(1 - \hat{p}_2)/n_2}.$$

Correlation

Under certain conditions the test statistic

$$T_\rho = \frac{R}{\sqrt{(1 - R^2)/(n-2)}}$$

has a t-distribution with $n - 2$ degrees of freedom. Here r is the sample linear correlation coefficient given by

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left[\frac{(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y} \right]$$

and is viewed as the realization of the random variable R.

Linear regression

Let β_0 be the unknown intercept and β_1 the unknown slope of a linear regression model with one explanatory variable, and let b_0 and b_1 be the corresponding estimators, i.e. the intercept and slope of the regression line (the 'best' line). Then b_0 and b_1 are given by

$$b_1 = r \frac{s_y}{s_x}$$

and

$$b_0 = \bar{y} - b_1 \bar{x}.$$

If certain requirements are met, then the observed value

$$t_\beta = \frac{b_1 - \beta_1}{s_{b_1}}$$

is a realization of the test statistic T_β that under the null hypothesis has a t-distribution with $n - 2$ degrees of freedom. Here s_{b_1} is the standard error (i.e. estimated standard deviation) of the estimator b_1.

NEGATIVE *z* Scores

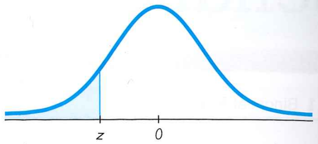


Table 2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	* .0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of z below 3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

z Score	Area
−1.645	0.0500
−2.575	0.0050

(*continued*)

POSITIVE *z* Scores

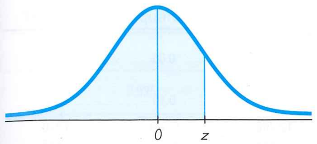


Table 2 *(continued)* Cumulative Area from the LEFT

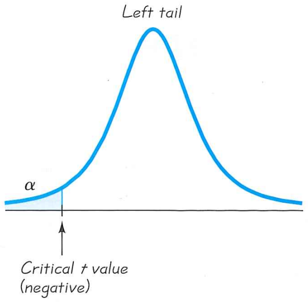
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	* .9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

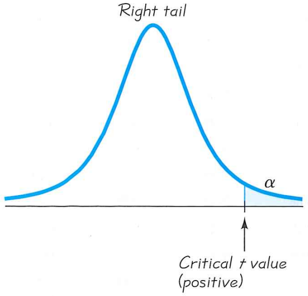
NOTE: For values of z above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

z Score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575





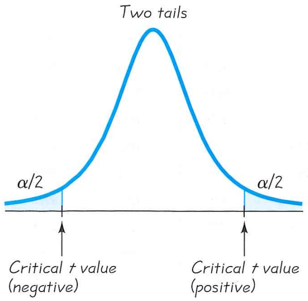


Table 3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319
24	**2.797**	2.492	**2.064**	1.711	1.318
25	**2.787**	2.485	**2.060**	1.708	1.316
26	**2.779**	2.479	**2.056**	1.706	1.315
27	**2.771**	2.473	**2.052**	1.703	1.314
28	**2.763**	2.467	**2.048**	1.701	1.313
29	**2.756**	2.462	**2.045**	1.699	1.311
30	**2.750**	2.457	**2.042**	1.697	1.310
31	**2.744**	2.453	**2.040**	1.696	1.309
32	**2.738**	2.449	**2.037**	1.694	1.309
33	**2.733**	2.445	**2.035**	1.692	1.308
34	**2.728**	2.441	**2.032**	1.691	1.307
35	**2.724**	2.438	**2.030**	1.690	1.306
36	**2.719**	2.434	**2.028**	1.688	1.306
37	**2.715**	2.431	**2.026**	1.687	1.305
38	**2.712**	2.429	**2.024**	1.686	1.304
39	**2.708**	2.426	**2.023**	1.685	1.304
40	**2.704**	2.423	**2.021**	1.684	1.303
45	**2.690**	2.412	**2.014**	1.679	1.301
50	**2.678**	2.403	**2.009**	1.676	1.299
60	**2.660**	2.390	**2.000**	1.671	1.296
70	**2.648**	2.381	**1.994**	1.667	1.294
80	**2.639**	2.374	**1.990**	1.664	1.292
90	**2.632**	2.368	**1.987**	1.662	1.291
100	**2.626**	2.364	**1.984**	1.660	1.290
200	**2.601**	2.345	**1.972**	1.653	1.286
300	**2.592**	2.339	**1.968**	1.650	1.284
400	**2.588**	2.336	**1.966**	1.649	1.284
500	**2.586**	2.334	**1.965**	1.648	1.283
1000	**2.581**	2.330	**1.962**	1.646	1.282
2000	**2.578**	2.328	**1.961**	1.646	1.282
Large	**2.576**	2.326	**1.960**	1.645	1.282

Table 4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Source: Donald B. Owen, *Handbook of Statistical Tables.*

Degrees of Freedom

$n - 1$	**Confidence interval or hypothesis test** for a standard deviation σ or variance σ^2
$k - 1$	**Goodness-of-fit** with k categories
$(r - 1)(c - 1)$	**Contingency table** with r rows and c columns
$k - 1$	**Kruskal-Wallis test** with k samples