**Resit Exam Sets and Combinatorics**
April 3, 2017, 18:30–20:30

This exam consists of 5 exercises with a total of 11 parts, for which you can obtain a maximum possible score of 36 points. The grade is computed as (number of points + 4)/4. You are not allowed to use aids such as calculators or notes. A brief and clear explanation should accompany all your answers.

**1**. In this exercise we consider the set-theoretic equality

$$B^c \cup (A \setminus C) = (B \setminus A)^c \cap (B \cap C)^c.$$

(a) [3 points] Use Venn-diagrams to show that this equality holds for all sets $A$, $B$ and $C$. Clearly illustrate how the sets on each side of the equality are constructed, by drawing a new Venn-diagram for every set operation used in the construction.

(b) [4 points] Now prove the above equality by using either the algebra of sets, or formal reasoning (your choice).

**2**. In the universe $U := \mathbb{R}$ consider the sets $A_n$ ($n \in \mathbb{N}$) defined by

$$A_n := \big[-\frac{1}{n}, n\big).$$

(a) [3 points] Determine a simple explicit expression for the set $\bigcap_{n=1}^{\infty} A_n$.

(b) [3 points] Determine a simple explicit expression for the set $\bigcap_{n=1}^{\infty} A_n^c$.

**3**. An urn contains 5 red balls numbered 1 through 5, 5 green balls numbered 6 through 10 and 5 blue balls numbered 11 through 15. In one grab, we draw 4 balls from the urn, and lay them in a row on a table in such a way that the numbers on the balls are in ascending order. We denote the outcome as an ordered sequence of the numbers (in ascending order).

(a) [3 points] What is the total number of possible outcomes?

(b) [3 points] How many outcomes are in the event that exactly two green balls were drawn?

(c) [3 points] How many outcomes are in the event that the second and the third ball in the row are both not green?

(see reverse side)

**4**. [5 points] Using mathematical induction, prove that for all $n \in \mathbb{N}$,

$$1 + \sum_{k=1}^{n} k \cdot (k!) = (n+1)!$$

**5**. Let $E$ be the set defined by $E := \{2n \colon n \in \mathbb{N}_0\}$, and let $f \colon \mathbb{Z} \to E$ be the function given by

$$f(n) = n + |n|.$$

(a) [3 points] Determine the image of the set $E$ under $f$.

(b) [3 points] Determine whether the function $f$ is injective.

(c) [3 points] Determine whether the function $f$ is surjective.

Do not forget to briefly explain your answers!