Examination Quantitative Financial Risk Management QF 4.5, May 27, 2010, 12:00 - 14:45.

Please provide motivated, concise and clear answers, in readable writing.
The exam grade is worth 55% of your final grade. Other 45% are composed as follows: 20% is the first assignment, 15% is the second assignment and 10% the presentation.
Results of the exam, the assignments and the presentations will be announced on June 10, 2010. Questions regarding your grades can be addressed to me by email after that date. The deadline for the second assignment is June 4, 2010.

1. You have 2 mln Euros invested into AEX and DJ indices in equal proportion. Using this as the collateral, you borrow another 2 mln Euros against the current 1-year German government bond interest rate, which is excessively low at the moment due to rescue package for Greece. You want to exploit this Greek situation: you saw that (because of obvious reasons) Greek bonds pay in excess of 15% at the moment, so you invest your borrowed 2 mln Euros into 2-year Greek government bonds (assume no coupons).

   a) Identity your risk factors. Write down the loss operator, which maps the risk factor changes into losses.

   b) Calculate a first-order approximation of the loss operator (i.e. write down the *linearized loss operator*), and express the portfolio loss as a linear function of the risk factor changes.

   c) Calculate the portfolio weights in b) if AEX today was at 310, DJ at 10065, Eurodollar exchange rate is at 1.25, German bond yield at 2.5% and Greek bond yield at 15%.

   d) Discuss how your investment strategy can go completely pear-shaped, i.e., explain possible scenarios leading to excessive loss.

2. A 1-mln Euros portfolio consists of stock index, commodity index and a bond, in equal proportions (1/3). The yearly average return is 12% for the stock index, 20% for commodity index and 5% for a bond. The following table gives an estimated historical Variance-Covariance matrix of the returns (all numbers expressed as yearly):

$$\begin{pmatrix} 0.1 & 0.04 & 0.03 \\ 0.04 & 0.2 & -0.04 \\ 0.03 & -0.04 & 0.6 \end{pmatrix}$$

   a) Compute the yearly and daily volatility of your portfolio and the average yearly portfolio return.

   b) Give the estimates of Value-at-Risk(0.95, 1 day) and VaR(0.99, 1 day), assuming normality of the returns and also assuming the returns have Student-t distribution with 4 degrees of freedom (the corresponding quantilies are 1.645 and 2.326 for Normal (0,1) and 2.132 and 3.747 for Student-t(4)).

   c) Under the assumption of Normal distribution, compute also the Expected Shortfalls corresponding to VaRs in b) and compute VaR(0.95, 5 days), VaR(0.95, 10 days) and VaR(0.95, 1 year).

3. The most recent volatility estimate for an index is 43% p/a. The today's return on the index is -0.5%. Using EWMA with the usual parameter 0.94, how does this latest return updates your volatility estimate? *Hint: do not forget to express everything either in daily vols or in yearly vols, assuming 251 trading days per year!*

4. a) Give a description of Merton's model.

   b) Use Merton's model to explain why there is a conflict of interest between shareholders and bondholders. Furthermore describe the effect of two parameters in Merton's model reflecting this conflict.

5. Consider a portfolio of 3 credit risks and assume a one factor Gaussian model. Assume that the probability of default of company k, with $k = 1, 2$ or 3 is given by $p_k(y)$. Model the default of company k by a latent variable $V(k)$ which has a standard normal distribution. Assume that the factor Y has a standard normal distribution.

   a) Describe the one-factor model.

   b) Give the probability of one default conditional on Y.

   c) Give a formula for the probability of one default.

6. Assume that the default time τ of a company is exponentially distributed with constant hazard rate $\lambda = 1$ and distribution function $F(t) = 1 - exp(-t)$ for all t. Furthermore assume a constant (continuously compounding) interest rate equal to 5% p/a and a deterministic loss given default equal to 60%. Consider a one-year CDS contract with spread s, with premium payments at $t = 1/2$ and $t = 1$.

   a) Give the value of the premium payment leg and default payment leg.

   b) Give an expression for the value of the fair swap spread (i.e., at which the CDS trades at par).