VU University Amsterdam, 2nd July 2014

At this exam, you may use copies of the slides without handwritten comments. Answers must be given in English. The exercises in this exam sum up to 90 points; each student gets 10 points bonus.

QUESTION 1. Given the sorts *Bool*, with the constructors $T, F :\to Bool$. Given also the sort D with non-constructor $eq : D \times D \to Bool$ and the sort *List* over D with constructors $[] :\to List$ and $in : D \times List \to List$.

Specify a non-constructor $insertNoDuplicate : D \times List \to List$ that, given a data item and an unordered list, returns the list with the data item inserted as the last element, only if it was not already in the list, otherwise the original list is returned unchanged.

You may use help functions, which must then also be defined. (8pts)

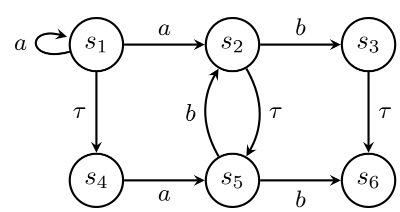
QUESTION 2. Consider the following μCRL specification:

$$Y = a.Z.Z$$
$$Z = a.Y + b$$

(a) Linearise the specification, using the algorithm underlying `mcrl`. (8pts)

(b) Would the algorithm underlying `mcrl -regular` terminate on this specification? Justify your answer. (4pts)

QUESTION 3. Consider the following state space:



(a) Calculate for each of the following two formulas, which states in the state space satisfy the formula. (Provide also the intermediate steps of your calculation.) (8pts)

(i) $\nu X.(\langle \neg a \rangle X)$
(ii) $\mu X.(\langle T \rangle T \wedge [\neg b] X)$

(b) State for each of the following two formulas which states in the state space satisfy the formula. (Explain your answer.) (8pts)
(i) $[T^*.a.a]F$
(ii) $\langle (\neg \tau)^* \rangle [T] F$

(c) Apply the minimization algorithm modulo branching bisimilarity to this state space. Describe the subsequent splits that you perform, and the results of those splits. Also draw the resulting minimized state space. (12pts)

Question 4. Let $even : Nat \to Bool$ with $even(n) = T$ if and only if n is an even number: $even(0) = T$ and $even(S(n)) = \neg(even(n))$.

(a) Consider the following LPE:

$$X(n : Nat) = \tau.X(n) \lhd even(n) \rhd \delta$$
$$+a(n).X(S(S(n))) \lhd T \rhd \delta$$

Give the confluence formula for the τ-summand, and argue that it is *true*, meaning that the τ-transitions generated by this summand are confluent. (10 pts)

(b) Consider the following LPE:

$$X(n : Nat) = \tau.X(S(n)) \lhd T \rhd \delta$$
$$+a.X(S(S(n))) \lhd even(n) \rhd \delta$$

Give the confluence formula for the τ-summand, and argue that it is *false*. Also give an example consisting of transitions of $X(0)$ to show that the τ-summand gives rise to non-confluent τ-transitions. (10 pts)

Question 5. *Leader Election Protocol:* Consider a ring network of processes. Communication is unidirectional: each process can only send messages to its clockwise neighbour. Each process has a unique identity, and there is a total ordering on the domain of identities. The process with the largest identity is elected as the leader as follows.

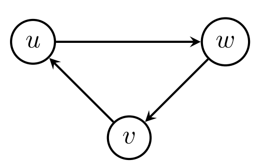
Each process is either active or passive; initially they are all active. An active process can send out one message containing its identity. If an active process with identity u receives a message carrying identity v, then:

- if $u < v$, the process becomes passive, and the message is forwarded;
- if $u > v$, the process remains active, and the message is cancelled;
- if $u = v$, the process becomes the leader, i.e., it performs an action $leader(u)$

Passive processes simply forward all incoming messages.

(a) Give a μCRL specification of a process in this leader election protocol, including the action declaration **act** and the communication declaration **comm**. (The data types that you use do not have to be formally specified.) Communication between processes is synchronous (i.e., you do not have to model channels as separate entities). (14 pts)

(b) Give the initial state of the following ring network of three processes, where u,v, and w refer to process identities. (Apply the encapsulation operator, but not the hiding operator.)



Let $u > v > w$. Draw the process graph belonging to this network. (5pts)

(c) Apply the hiding operator to rename all communications into τ. Draw the process graph for the network above after minimization modulo branching bisimilarity. (3pts)