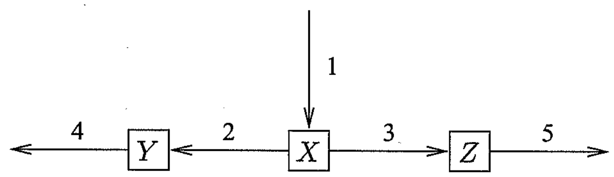
Resit Protocol Validation

Free University Amsterdam, 9 February 2010, 18:30-21:15

(At this exam, you may use copies of the slides without handwritten comments. Answers can be given in English or Dutch.)

(The exercises in this exam sum up to 90 points; each student gets 10 points bonus.)

1. Data elements from a set Δ can be received by X via channel 1 (by $r_1(d)$), after which they are *alternernatingly* sent on to Y via channel 2 (by $s_2(d)$) or to Z via channel 3 (by $s_3(d)$). So the first received datum is sent to Y, the second to Z, the third to Y, etc. Y sends on received data elements via channel 4, while Z sends on received data elements via channel 5.



   (a) Specify the parallel processes X, Y and Z in μCRL, including the action declaration **act** and communication declaration **comm**. (The data types that you use do not have to be specified.) (8 pnts)

   (b) Let $\Delta = \{0\}$. Draw the process graph of $\partial_{\{r_2,s_2,r_3,s_3\}}(X\|Y\|Z)$. (6 pnts)

   (c) Draw the process graph of $\tau_{\{c_2,c_3\}}(\partial_{\{r_2,s_2,r_3,s_3\}}(X\|Y\|Z))$ after minimisation modulo branching bisimilarity. (4 pnts)

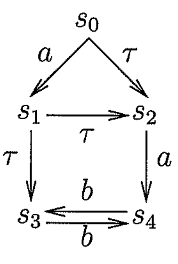
2. Linearise, using the algorithm underlying `mcrl`, the μCRL specification

$$\begin{aligned} Y(m{:}Nat) &= a(m)\cdot Z(S(m))\cdot Y(m) \\ Z(m{:}Nat) &= b(m)\cdot Z(S(m)) + c(m) \end{aligned}$$

   with initial state $Y(0)$.

   Would the algorithm underlying `mcrl -regular` terminate on this specification? Explain your answer. (12 pnts)

3. Consider the following process graph.



(a) What is the maximal collection of confluent τ-transitions in this process graph? Explain your answer. (8 pnts)

(b) Apply the minimisation algorithm modulo branching bisimilarity to this process graph. Describe the subsequent splits that you perform, and the results of those splits. Also draw the resulting minimised process graph. (12 pnts)

(c) Compare a reduction based on confluent τ-transitions with applying the minimisation algorithm modulo branching bisimilarity. What are the advantages and disadvantages of both approaches? (4 pnts)

4. (a) Given functions $\mathit{even} : \mathit{Nat} \to \mathit{Bool}$ and $\mathit{div3} : \mathit{Nat} \to \mathit{Bool}$ with:

   - $\mathit{even}(n) = \mathsf{T}$ if and only if n is an even number;
   - $\mathit{div3}(n) = \mathsf{T}$ if and only if n is divisible by three.

   Consider the following μCRL specification:

$$\textbf{proc}\quad X(n{:}\mathit{Nat}) \;=\; a(\mathit{div3}(n))\cdot X(S(S(S(n))))$$

   Is *even* an invariant for X? Is *div3* an invariant for X? If yes, give a proof. If no, give a counter-example. (4 pnts)

   (b) Let $Y = a(\mathsf{T})\cdot Y$. Prove, using CL-RSP, that $X(n) = Y$ for numbers n that are divisible by three. (8 pnts)

5.

$$\begin{array}{rcl} X(m{:}\mathit{Nat}) & = & a(m)\cdot X(S(m)) \lhd \mathit{even}(m) \rhd \delta \\ & + & \tau\cdot X(S(m)) \lhd \neg \mathit{even}(m) \rhd \delta \end{array}$$

$$Y(n{:}\mathit{Nat}) \;=\; a(n)\cdot Y(S(S(n)))$$

Show that X is convergent. What is the focus condition for X? Give a state mapping $\phi : \mathit{Nat} \to \mathit{Nat}$ such that the matching criteria are satisfied with respect to X and Y. Prove that the matching criteria are satisfied indeed. (12 pnts)

6. Consider the state space of the μCRL specification $\partial_H(S(0) \parallel R(0) \parallel K \parallel L)$ of the Alternating Bit Protocol, where H consists of all send and read actions over the internal channels B, C, E and F (see the slides). Let $\Delta = \{d, e\}$.

   Consider the following abstraction. All communication actions over the internal channels and j are mapped to a special action $\hat{c}$, while the actions $r_{\mathrm{A}}(d)$, $r_{\mathrm{A}}(e)$, $s_{\mathrm{D}}(d)$ and $s_{\mathrm{D}}(e)$ are left in tact. The abstracted state space has three states: empty, d and e, where each state in the concrete state space is mapped to the datum that is currently being transported, or empty if no datum is being transported.

   Give precise definitions of the mappings π and θ, and draw the state space of the abstracted Alternating Bit Protocol. (12 pnts)