Short Test 2 – Probability Theory course 2023

Time available: 30 minutes

The short test grade is given by $1+$ the number of points (thus 10 is the maximum), rounded to the 1st decimal.

Exercise 1 [3 points]
Consider a continuous random variable X with probability density function

$$f_X(x) := \begin{cases} c \cdot (4x - 2x^2) & \text{if } 0 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [1 point] Find the correct value of c.

(b) [1 point] Give the corresponding cumulative distribution function F_X.

(c) [0.5 point] Calculate $\mathbb{P}\left(\dfrac{1}{2} < X < \dfrac{3}{2}\right)$.

(d) [0.5 point] Calculate $\mathbb{P}\left(X = \dfrac{2}{3}\right)$ and $\mathbb{P}\left(X = \dfrac{5}{6}\right)$.

Exercise 2 [6 points]
Jasmine has three children, each of which is equally likely to be a boy or a girl, independently of the others. Consider the events:

$$A = \{\text{all the children are of the same sex}\}$$
$$B = \{\text{there is at most one boy}\}$$
$$C = \{\text{the family includes at least a boy and a girl}\}$$

(a) [1.75 points] Show that A is independent of B and that B is independent of C.

(b) [0.5 point] Is A independent of C? Justify your answer extensively in either case.

Assuming now that Jasmine has still three children, but the probability of each child being a girl is equal to $\frac{3}{5}$, independently of the others. Let X be the number of Jasmine's daughters.

(c) [0.75 points] What is the distribution of X? Give the probability mass function of X.

(d) [1 point] Consider the events B and C as defined above and express them using the random variable X. Is B independent of C? Justify your answer extensively in either case.

Jasmine has 11 male cousins and 9 female cousins. During a family gathering where there all present, 6 out of these 20 cousins are chosen at random to form a volleyball team. Denote by Y the number of female players in such a team.

(e) [1 point] What is the distribution of Y? Specify the probability mass function of Y.

(f) [1 point] What is the probability that the team consists of at least 5 female players?