Mock Short Test 2

Time available: 45 minutes

Exercise 1

Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} 2(x-1) & \text{if } 1 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

Find the cumulative distribution function $F_X(x)$ of the random variable X for all $x \in \mathbb{R}$

Exercise 2

Consider an urn with 3 red and 7 blue balls.

(a) Mary takes 2 arbitrary balls from this urn, *without replacement*. Let X be the number of blue balls that she obtains. Give the probability mass function of X and the name of its distribution.

(b) We put the balls that Mary took back, so we are back with an urn with 3 red and 7 blue balls. We now let Boris take a ball from this urn five times, *with replacement*. Let Y be the total number of times Boris obtains a blue ball. Give the probability mass function and the name of the distribution of Y.

(c) Finally, Mo takes balls from the same urn with 3 red and 7 blue balls, *with replacement*, and continues until he takes a red ball. Let W be the number of balls Mo needs to draw until he succeeds. Give the probability mass function and the name of the distribution of W.

Exercise 3

Suppose we flip a fair coin 3 times. Let A be the event that we obtain heads at most once and let B be the event that we obtain heads at least once and tails at least once. Are A and B independent?

Exercise 4

We have an urn with 3 red balls and 2 blue balls. We pick a sample of three balls *without replacement* and put these balls in a second urn that was previously empty. Then we sample three balls from the second urn *with replacement*. Let X be the number of blue balls taken from the first urn and let Y be the number of blue balls taken from the second urn. Compute $\mathbb{P}(X = 1 \mid Y = 2)$.