Short Test Probability Theory, May 13, 2022, 16:30-17:15

Name:

Student number:

Please write your answers on this paper. You do not need a calculator for this test (although a simple calculator is allowed). Explain your answers carefully! Your grade is given by (1+ number of points obtained).

Exercise 1. [3 points] Let X and Y be independent uniform random variables on (1,5). Find their joint density function $f(x, y)$. Use the joint density function to calculate the probability $P(X < Y)$.

Exercise 2. [4 points] Suppose that X and Y are independent continuous random variables with density functions

$$f_X(x) = \begin{cases} 3xe^{-2x} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$f_Y(y) = \begin{cases} 2ye^{-2y} & \text{if } y \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find the density function of $Z := X + Y$.

Exercise 3. [2 points] Let X and Y be independent normal random variables with $X \sim N(1, 3)$ and $Y \sim N(-1, 4)$. Identify the distribution of $W := 2X + 5Y$.