· Short test probability theory April 29 2021

1 a) $f_Z(z) = 0$ for $z < 1$

for $z \geq 1$:

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx$$

$$= \int_{-1}^{\infty} e^{-x-1} f_Y(z-x)\, dx$$

· $-x \geq 2$

$-x \geq 2-z$

$x \leq z-2$

$$= \int_{-1}^{z-2} e^{-x-1} e^{-(z-x)+2}\, dx$$

$$= \int_{-1}^{z-2} e^{-z+1}\, dx = \Big[x e^{-z+1} \Big]_{x=-1}^{x=z-2} = (z-1)e^{-z+1}$$

$z \geq 1$

Or $f_t(t) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y)\, dy$

$= \int_{2}^{\infty} f_X(z-y)\, e^{-y+2}\, dy =$

$z - y \geq -1$

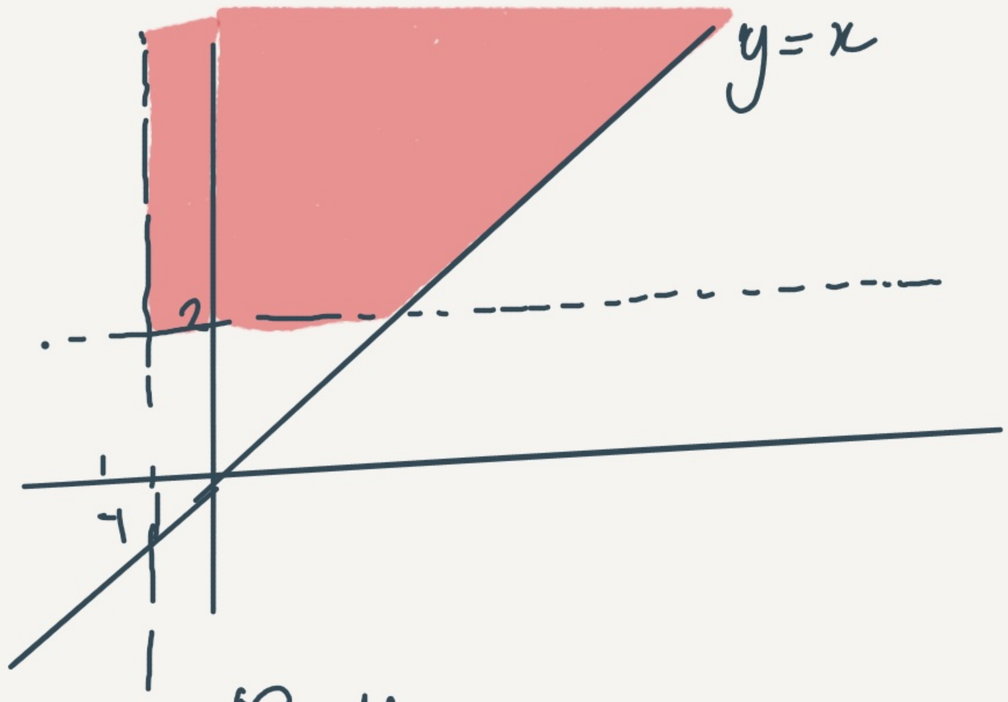
$-y \geq -1 - z$

$y \leq z+1$

$= \int_{2}^{z+1} e^{-(z-y)-1} e^{-y}\, dy$

$= \int_{2}^{z+1} e^{-z+1}\, dy = \left[y e^{-z+1} \right]_{y=2}^{y=z+1} = (z-1) e^{-z+1}$

b)

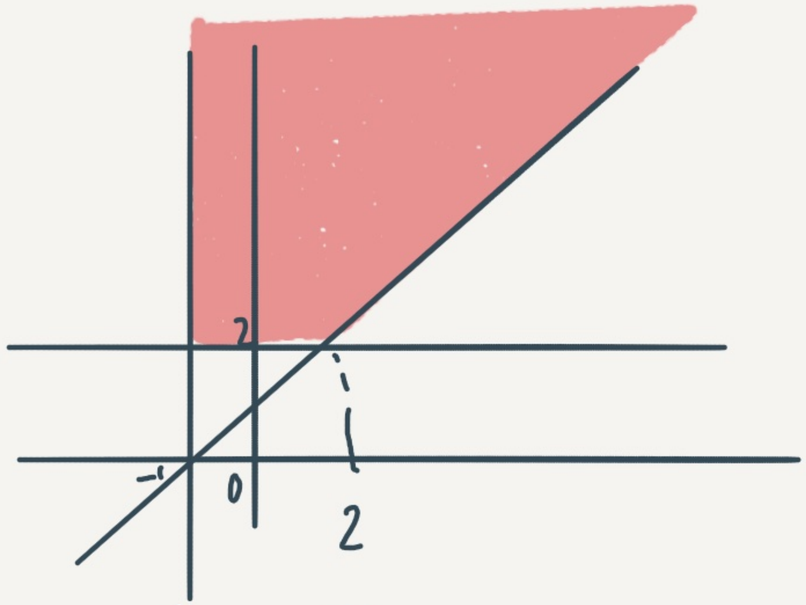


$$P(Y>X) = \int_{2}^{\infty}\int_{-1}^{y} e^{-x-1}\cdot e^{-y+2}\,dx\,dy$$

$$= \int_{2}^{\infty} e^{-y+2}\cdot\left[-e^{-x-1}\right]_{x=-1}^{x=y} dy = \int_{2}^{\infty} e^{-y+2}\left(-e^{-y-1}+1\right)dy$$

$$= \int_{2}^{\infty}\left(-e^{-2y+1}+e^{-y+2}\right)dy = \left[\frac{1}{2}e^{-2y+1} - e^{-y+2}\right]_{2}^{\infty} = -\frac{1}{2}e^{-3}+1$$

Or



$$P(Y>X) = \int_{-1}^{2}\int_{2}^{\infty} e^{-x-1}\, e^{-y+2}\, dy\, dx + \int_{2}^{\infty}\int_{x}^{\infty} e^{-x-1}\, e^{-y+2}\, dy\, dx$$

$$= \int_{-1}^{2} e^{-x-1}\underbrace{\left[-e^{-y+2}\right]_{2}^{\infty}}_{1} dx + \int_{2}^{\infty} e^{-x-1}\left[-e^{-y+2}\right]_{x}^{\infty} dx$$

$$= \left[-e^{-x-1}\right]_{-1}^{2} + \int_{2}^{\infty} e^{-x-1}\left(e^{-x+2}\right) dx =$$

$$= -e^{-3} + 1 + \int_{2}^{\infty} e^{-2x+1}\, dx = e^{-3} + 1 + \left[-\frac{1}{2}e^{-2x+1}\right]_{2}^{\infty} = -e^{-3} + 1 + \frac{1}{2}e^{-3}$$

$$= 1 - \frac{1}{2}e^{-3}.$$

③ W is normally distributed with expectation $3.0 - 4.0 = 0$ and variance $9 + 16 = 25$

$$W \sim N(0, 25)$$