Short Test Probability Theory, April 22, 2022, 16.30-17.15

Name:

Student number:

Please write your answers on this paper. You may use a simple calculator for this test, but this is not necessary. Your grade is given by 1 + number of points. Explain your answers carefully!

Exercise 1. [3 points] Let X be a continuous random variable with density

$$f_X(x) = \begin{cases} \frac{4}{x^5} & \text{if } x \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the density function of $Y := \ln(X)$.

Exercise 2. [3 points] We toss a fair coin 200 times. Let X be the number of times that the coin lands with heads up. Give an approximation of $P(X \leq 104)$ that is based on the central limit theorem, don't forget the continuity correction. Express your answer in terms of the cumulative distribution function Φ of a standard normal random variable.

Exercise 3. [3 points] X and Y are continuous random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} e^{-y} & \text{if } x \geq 0 \text{ and } y \geq x, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the marginal density function of Y.