Midterm Exam Probability Theory

March 28, 2022, 12.15-14.15

- This midterm exam consists of four questions and a table. You can score 36 points. Your grade is given by (4+number of points)/4.
- You may use a simple calculator, but a graphical or programmable calculator is not allowed.
- Explain your answers clearly.

1. An urn contains one red, one yellow and one blue ball. Take 3 balls from this urn with replacement. Let A be the event that you obtain at least one red ball, let B be the event that you obtain at least one blue ball, and let X be the number of different colours that you obtain.
(a) [4 points] Compute $E(X)$.
(b) [3 points] Compute $P(A \cup B)$.
(c) [3 points] Compute $P(A|B)$.
(d) [2 points] Are A and B independent?

2. A company produces chips for integrated circuits. These chips are defective with probability 0.05, independently of each other. There is a test to check these chips, which is not entirely reliable. The conditional probability that a chip is declared defective given that it is defective is 0.95, and the conditional probability that a chip is declared sound (that is: not defective) given that it is sound is 0.97. Test results of different chips are independent of each other.
(a) [4 points] A chip is tested and declared defective. Compute the conditional probability that this chip is sound.
(b) [3 points] John will test 10 chips. Let X be the number of these chips that are declared sound. Give the probability mass function of X.
(c) [3 points] Mary will test chips one by one, until she obtains a chip that is declared defective. Let Y be the number of chips she tests. Give the name of the probability distribution of Y, the probability mass function of Y and $E(Y)$.

Please turn over!

3. The continuous random variable X has density function

$$f_X(x) = \begin{cases} tx^4 & \text{if } -1 \leq x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where t is a constant.
(a) [2 points] Compute the constant t.
(b) [3 points] Let F_X be the cumulative distribution function of X. Compute $F_X(x)$ for all $x \in \mathbb{R}$.
(c) [3 points] Compute $\text{Var}(X)$.
(d) [3 points] Compute $E(|X|)$.

4. [3 points] Let X be a normal random variable with mean 2 and $P(X > -1.5) = 0.975$. Use the table to determine the variance of X.