Midterm Exam Probability Theory

March 22, 2021, 12.15-14.15 (extra time 12.15-14.45)

- This exam consists of five questions and a table. You can score 36 points. Your grade is given by (4+number of points)/4.
- You may use a simple calculator, but a graphical or programmable calculator is not allowed.
- Explain your answers clearly.
- You have to keep track of the time yourself. Make sure that you stop at 14.15 (or 14.45 if you have extra time) and upload your answers as one single pdf via Canvas within 10 minutes.
- If you want to stop before the end time of the exam, ask permission of your TA via the chat. After the TA gives you permission, upload your answers as one single pdf via canvas within 10 minutes and additionally let the TA know in the chat when you finished uploading. You can then leave the meeting, but it is not allowed to have any communication with other students about the exam before 15.15.
- If you need to go to the bathroom, let the TA know in the chat and wait for permission. If you don't get permission within 2 minutes, just go and mention when you are back.
- It is of great help if you start each of the five exercises on a new page!
- Good luck!

1. We roll five fair dice.
(a) [3 points] Let X be the number of dice on which the value '4' appears. Compute $P(X \geq 2)$.
(b) [3 points] Compute the probability that we see a 'large straight', that is, we see either the values 1, 2, 3, 4 and 5 or the values 2, 3, 4, 5 and 6.

2. We have two urns. The first urn contains three balls labelled 1,2 and 3 and the second urn contains four balls, labelled 2, 3, 4 and 5.
(a) [4 points] Jeremy picks a random ball from the first urn and moves this ball to the second urn. Then he takes a random ball from the second urn. Let X be the label of this ball. Compute $\mathrm{Var}(X)$.
(b) [3 points] Now we put all the balls together in urn 2, so urn 2 now contains one ball labeled 1, two balls labelled 2, two balls labelled 3, one ball labelled 4 and one ball labelled 5. Mary samples balls from this urn repeatedly with replacement, until she obtains the ball labelled 5. Let Y be the number of balls Mary samples. Compute $P(3 \leq Y < 10)$.

3. The help desk of company HLQ is run by three employees: Mr. H., Mr. L. and Mrs. Q. 45% of all service requests at the help desk are handled by Mr. H., 30% are handled by Mr. L. and 25% are handled by Mrs. Q. After a service request is handled, the customer rates the quality of the delivered service, where the customer can choose between the ratings good, sufficient and insufficient. In the table below you can find for each employee the percentages of how the customer rates the quality of the delivered service of that employee.

Employee	Good	Sufficient	Insufficient
Mr. H.	90%	3%	7%
Mr. L.	85%	10%	5%
Mrs. Q.	76%	18%	6%

We randomly select a service request handled at the help desk and introduce the following events:
H: the request is handled by Mr. H.,
L: the request is handled by Mr. L.,
Q: the request is handled by Mrs. Q.,
G: the quality of the delivered service is rated as good,
S: the quality of the delivered service is rated as sufficient,
I: the quality of the delivered service is rated as insufficient.

(a) [3 points] Compute $P(Q \cup S)$.
(b) [3 points] Compute $P(H \mid I)$.
(c) [3 points] Are L and G independent?

4. Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} c(1-x)x^2 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [2 points] Compute the constant c.
(b) [3 points] Let F_X be the cumulative distribution function of X. Compute $F_X(x)$ for all $x \in \mathbb{R}$.
(c) [3 points] Compute $E(2X^2 + X)$.

5. Let X be a normal random variable with mean μ and variance σ^2.
(a) [3 points] Use the table to compute $P(|X - \mu| > 2\frac{1}{2}\sigma)$.
(b) [3 points] If $\mu = 1$ and $\sigma^2 = 4$, use the table to compute c such that $P(X > c) = 0.975$.

Appendix E

Table of values for $\Phi(x)$

$\Phi(x) = P(Z \leq x)$ is the cumulative distribution function of the standard normal random variable Z.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998