Short Test Probability Theory, March 11, 2021

16.30-17.15 (extra time: 16.30-17.25)

- Please write your answers on a sheet of paper, mention your name and student number. After the test, upload your answers as one single pdf via Canvas within 10 minutes.
- You may use a simple calculator for this test, but this is not necessary. You don't have to work out powers, fractions, binomial coefficients, products, etc.
- You have to keep track of the time yourself. Make sure that you stop at 17.15 (or 10 minutes later if you have extra time).
- If you want to stop before the end of the test, ask permission of the TA via the chat. Upload your answers within 10 minutes and let the TA know in the chat when you have finished uploading.
- Your grade is given by 1+ number of points.
- Explain your answers clearly, use notation and explanation, don't just write numbers!

Exercise 1
Let X be a continuous random variable with density function

$$f(x) = \begin{cases} 2(x-1) & \text{if } 1 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [2 points] Compute the expectation of X.
(b) [2 points] Find the cumulative distribution $F(x)$ of X (for all $x \in \mathbb{R}$).

Exercise 2
We have an urn with 3 red and 7 blue balls.
(a) [2 points] Mary takes 2 arbitrary balls from this urn, *without replacement.* Let X be the number of blue balls that she obtains. Compute $E(X)$.
(b) [$1\frac{1}{2}$ points] We put the balls that Mary took back in the urn, so we still have an urn with 3 ted and 7 blue balls. We now let Boris take a ball from this urn five times, *with replacement.* Let Y be the total number of times Boris obtains a blue ball. Give the probability mass function and the name of the distribution of Y.
(b) [$1\frac{1}{2}$ points] Finally, Mo takes balls from the same urn with 3 red and 7 blue balls, *with replacement*, and continues until he takes a red ball. Let W be the number of balls Mo needs to select and give the probability mass function and the name of the distribution of W.