First short test, Feb 25

1 a) $P(\text{at least 2 black socks}) =$
$P(\text{at least 2 black socks and drawer 1}) +$
$P(\text{at least 2 black socks and drawer 2})$

$= P(\text{drawer 1})\, P(\text{at least 2 black} \mid \text{drawer 1}) +$
$P(\text{drawer 2})\, P(\text{at least 2 black} \mid \text{drawer 2})$

$= P(\text{drawer 1})\, P(\text{two black and one white} \mid \text{dr 1})$
$+ P(\text{drawer 1})\, P(\text{three black socks} \mid \text{drawer 1})$
$+ P(\text{drawer 2})\, P(\text{two black and one white} \mid \text{dr 2})$

$$= \frac{2}{3} \frac{\binom{3}{2}\binom{4}{1}}{\binom{7}{3}} + \frac{2}{3} \cdot \frac{\binom{3}{3}}{\binom{7}{3}} + \frac{1}{3} \cdot \frac{\binom{2}{2}\binom{5}{1}}{\binom{7}{3}}$$

$$= \frac{2}{3} \cdot \frac{3 \cdot 4}{35} + \frac{2}{3} \cdot \frac{1}{35} + \frac{1}{3} \cdot \frac{5}{35} = \frac{31}{105}.$$

$$P(\text{first drawer} \mid \text{all same colour})$$

Bayes

$$\frac{P(\text{first drawer})\,P(\text{all same} \mid 1^{st}\text{ drawer})}{P(1^{st}\text{ drawer})\,P(\text{all same} \mid 1^{st}) + P(2^{nd}\text{ dr})\,P(\text{all same} \mid 2^{nd})}$$

$$= \frac{\frac{2}{3} \cdot \frac{\binom{4}{3}+\binom{3}{3}}{\binom{7}{3}}}{\frac{2}{3} \cdot \frac{\binom{4}{3}+\binom{3}{3}}{\binom{7}{3}} + \frac{1}{3} \cdot \frac{\binom{5}{3}}{\binom{7}{3}}}$$

$$= \frac{\frac{2}{3} \cdot \frac{4+1}{35}}{\frac{2}{3} \cdot \frac{5}{35} + \frac{1}{3} \cdot \frac{10}{35}} = \frac{\frac{10}{105}}{\frac{20}{105}} = \frac{1}{2}$$

Exercise 2

(a) $P(A) = 0{,}4 \quad P(B) = 0{,}3 \quad P(A \cup B) = 0{,}5$.

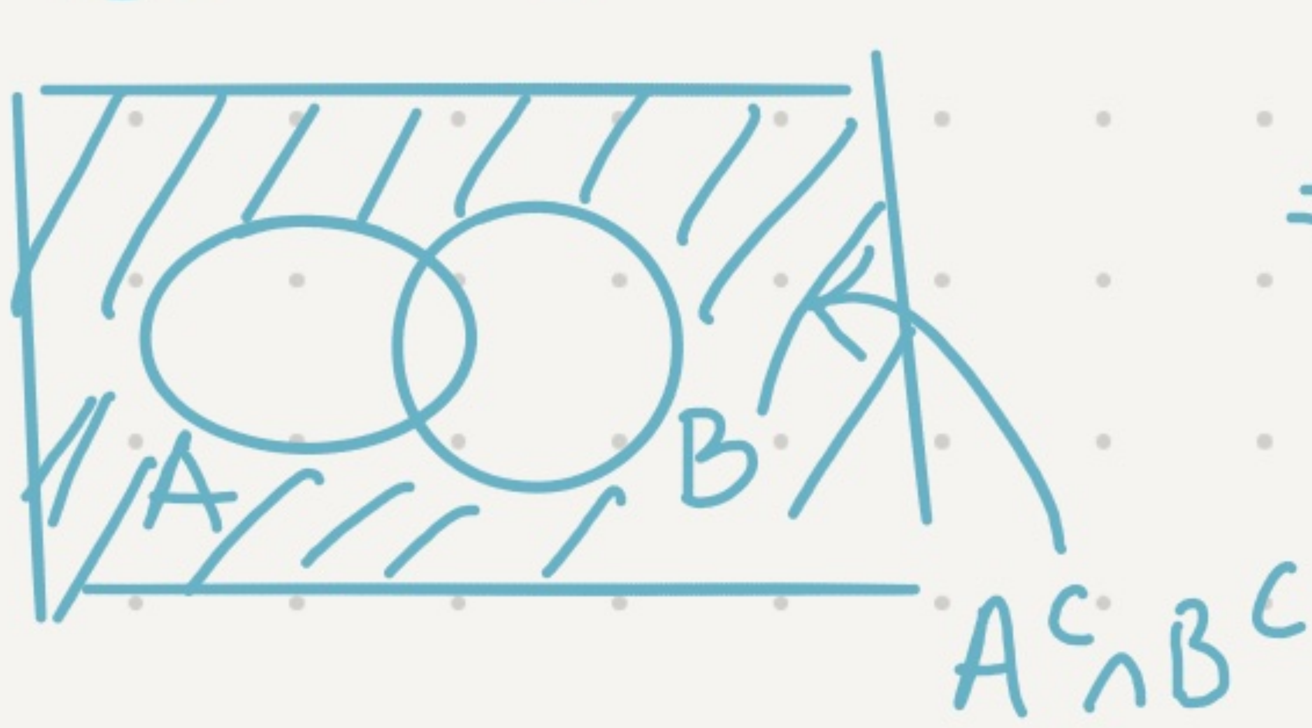
A and B are independent if
$P(A \cap B) = P(A)P(B)$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

This gives $0{,}5 = 0{,}3 + 0{,}4 - P(A \cap B)$ so
$P(A \cap B) = 0{,}2$.

$0{,}2 \neq 0{,}4 \cdot 0{,}3$ so A and B are dependent.

(b) $P(A^c \cap B^c) = 1 - P((A^c \cap B^c)^c)$



$= 1 - P(A \cup B)$

$= 1 - 0{,}5 = 0{,}5$.