Midterm Exam Probability Theory

March 25, 2019, 12.00-14.00

- This midterm exam consists of five questions and a table. You can score 36 points. Your grade is given by (4+number of points)/4.
- You may use a simple calculator, but a graphical or programmable calculator is not allowed.
- Explain your answers clearly.

1. A 3-person basketball team consists of a guard, a forward and a center. A person is chosen at random from each of three of those teams.
(a) [3 points] Compute the probability that the three selected players do not form a complete team.
(b) [3 points] Compute the probability that all selected players play the same position.
(c) [3 points] Let A be the event that at least two of the three selected players play the same position and B the event that at least one of the three selected players is a center. Compute $P(A \cup B)$.

2. [3 points] You have five coins coloured red, blue, white, yellow and orange. Apart from the variation in colour, the coins look identical. One of the coins is unfair and when tossed it comes up heads with probability $\frac{3}{4}$. The other coins are fair. You have no further information about the coins, so a priori you can assume that every coin has probability $1/5$ to be the unfair coin.
Suppose that you take the blue coin, toss it three times, and that on all three tosses heads comes up. Given this event, compute the conditional probability that the blue coin is the unfair one.

3. Two athletic teams play a series of at most 5 games. Suppose that one of the teams is stronger than the other and wins each game with probability 0.6 independent of the outcomes of the other games (the other team wins each game with probability 0.4; there is no possibility of a draw). The first team to win 3 games is declared the overall winner, and as soon as one of the teams has won 3 games, no additional games are played. Let X be the number of games that are played.
(a) [3 points] Show that $P(X = 3) = 0.28$, $P(X = 4) = 0.3744$ and that $P(X = 5) = 0.3456$.
(b) [3 points] Compute the variance of X.
(c) [3 points] Would it be more profitable for the weaker team if a series of at most 3 games is played and the first team to win 2 games is declared the overall winner? Support your answer with a calculation.

Please turn over!

4. The lifetimes of computer chips produced by a certain manufacturer are normally distributed with expectation $\mu = 14 \times 10^5$ hrs and standard deviation $\sigma = 3.0 \times 10^5$ hrs.
(a) [3 points] For this question you need the table. Show that the probability that a chip has a lifetime that is less than 7×10^5 hrs is approximately 0.010.
(b) [3 points] Consider a batch of 200 of these chips and let Y be the number of chips that have a lifetime less than 7×10^5 hrs. Give an approximation of $P(Y \leq 2)$ by approximating a binomial distribution by a Poisson distribution.

5. Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} 4x & \text{if } 0 \leq x < \frac{1}{2}, \\ 4 - 4x & \text{if } \frac{1}{2} \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [3 points] Show that f_X is indeed a density function and compute the cumulative distribution function $F_X(x)$ for all $x \in \mathbb{R}$.
(b) [3 points] Compute the expectation of X.
(c) [3 points] Compute $P\left(\frac{X}{1-X} > 4\right)$.

Table

Area ϕ(x) under the standard normal curve to the left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998