Midterm Probability Theory

March 29, 2018, 12.00-14.00

- This midterm consists of six questions and a table. You can obtain 36 points. Your grade is given by (4+number of points)/4.
- You may use a simple calculator, but it is not allowed to use a graphical or a programmable calculator.
- Explain your answers clearly.

1. We have a drawer with $n \geq 4$ socks. Three of these socks are red and the remaining ones are black. We choose 2 socks randomly, without replacement.
(a) [3 points] There is one value of n so that the probability that both socks are red is equal to $\frac{1}{2}$. Which value of n is that?

In the rest of this question we take $n = 7$.
(b) [3 points] Let X be the number of different colors we obtain. Compute $\text{Var}(X)$.
(c) [3 points] Let A be the event that that both socks have the same color and B the event that at least one of the socks is red. Compute $P(A|B)$.

2. [3 points] On the island of liars, each inhabitant lies with probability $\frac{2}{3}$. You overhear an inhabitant making a statement. Next you ask another inhabitant whether the inhabitant you overheard spoke truthfully. Compute the conditional probability that the inhabitant you overheard spoke truthfully given that the other says so.

3. Suppose that it takes at least 10 votes from a 12-member jury to convict a defendant. Suppose also that the (conditional) probability that a juror votes a guilty person innocent is 0.2, whereas the (conditional) probability that a juror votes an innocent person guilty is 0.1. Suppose also that each juror acts independently and that 65% of the defendants are guilty.
(a) [3 points] Compute the probability that the jury renders a correct decision.
(b) [3 points] Compute the probability that a randomly chosen case leads to a conviction.

4. Suppose that A and B are events for which $P(A^c) = 0.3$, $P(B^c) = 0.4$ and $P(A \cup B) = 0.88$.
(a) [2 points] Compute the probability that A occurs but B does not.
(b) [2 points] Are A and B independent?

Please turn over!

5. Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} cx(1-x) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [2 points] Compute c.
(b) [3 points] Compute the cumulative distribution function $F_X(x)$ of X for all $x \in \mathbb{R}$.
(c) [3 points] Compute $\text{Var}(3X - 7)$.

6. For this question you need the table. The diameter of a 1 euro coin is normally distributed with an expectation of 23.25 mm and a standard deviation of 0.10 mm. A vending machine accepts only 1 euro coins with a diameter between 22.95mm and 23.45mm.
(a) [3 points] Compute the probability that a euro coin is not accepted by the vending machine.
(b) [3 points] Let X be the diameter (in mm) of a 1 euro coin. Compute for which value d we have

$$P(|X - 23.25| < d) \approx 0.95.$$

Table

Area ϕ(x) under the standard normal curve to the left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998