Midterm Probability Theory

March 24, 2016, 12.00-14.00

- This midterm consists of six exercises and a table. You can obtain 36 points. Your grade is given by (4+number of points)/4.
- You may use a simple calculator, but it is not allowed to use a graphical or a programmable calculator.
- Explain your answers clearly.

1. Four people, named A, B, C and D, are randomly arranged on a line.
(a) [3 points] We assign number 1 to the first person on the line, number 2 to the second person etc. What is the probability that D obtains a higher number than both A and B?
(b) [3 points] Let S be the event that A and B are next to each other, and let T be the event that C is on the last position. What is $P(T|S)$?
(c) [4 points] Let Y be the number of people between C and D. Compute $E(Y)$.

2. A man has five coins in his pocket. Two are double-headed coins, two are ordinary coins and one coin is double-tailed.
(a) [4 points] He selects a random coin and tosses it, and the coin lands tails up. Compute the conditional probability that he selected an ordinary coin.
(b) [3 points] The man gives his five coins to a woman, who randomly selects three random coins from these five coins (without replacement). Compute the probability that at least one of these coins is double-headed.

3. [3 points] The probability of being dealt a full house in a hand of poker is approximately 0.0014. Give an approximation based on a Poisson distribution of the probability that in 2000 hands of poker you will be dealt exactly three full houses.

4. [4 points] Mary and Hugo play the following game. Mary gives Hugo three fair coins of one euro. Hugo flips these coins and keeps the coins landing heads up and has to return those landing tails up. Moreover, if all three coins land tails up, Hugo must pay Mary eight euros (in addition, besides returning Mary's coins). Let X be Hugo's net gain. Compute $E(X)$.

5. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{x}{6} & \text{if } 2 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [3 points] Compute the cumulative distribution function of X.
(b) [3 points] Compute $\mathrm{Var}(X)$.
(c) [3 points] Consider a right-angled triangle of which the hypotenuse has length 9 and one of the legs has length X. Compute the expected length of the other leg.

6. [3 points] Let X be a normal random variable with expectation 6 and variance 16. Use the table to compute the value c such that $P(X > c) \approx 0.025$.

Table

Area ϕ(x) under the standard normal curve to the left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998