Faculty of Sciences
(Use capitals / Write clearly)

#points=36

Result: 10

vrije Universiteit amsterdam

Initials and last name: [redacted]

Course: PT

Date: 24 maart '16

Studentnumber: [redacted]

Name of study: BA

1 a) total possibilities $= 4!$

possibilities $D > A \cap D > B$: AB DC, A B CD, 6x D

$P(D=3,\ A$ en $B = 1$ en $2)$
$= 2!/4!$ (1)

(3)

$P(D=4) = 3!/4!$ (1)

$P(D>A \cap D>B) = \frac{2!}{4!} + \frac{3!}{4!} = \frac{8}{24} = \frac{1}{3}$ (1)

b) $P(T) = 1/4$

AB of BA; 3 plekken waar op het kan; andere plekken CD of DC

$P(S) = \frac{2 \cdot 3 \cdot 2}{4!}$

$= \frac{12}{24} = \frac{1}{2}$ (1)

(3)

$P(TS) = \frac{2 \cdot 2 \cdot 1}{4!} = \frac{4}{24} = \frac{1}{6}$ (1)

T & S afhankelijk

dan $P(T \mid S) = \frac{1/6}{1/2} = 1/3$ (1)

c) $P(Y=0) = P(S) = 1/2$ (1½) (2 plekken waar ze kunnen staan en CD of DC en AB of BA)

$P(Y=1) = \frac{8}{4!} = \frac{8}{24} = 1/3$

$P(Y=2) = \frac{4}{4!} = \frac{4}{24} = 1/6$ (1) (1 plek & ACD of DC & AB of DA)

$= 1$ → klopt!

(4)

$E(Y) = \sum_{i=0}^{2} i\, P(Y=i)$

$= 0 \cdot 1/2 + 1 \cdot 1/3 + 2 \cdot 1/6 = 2/3 = E(Y)$ (1½)

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6 $P(X > c) = P\left(\frac{X-\mu}{\sigma} > \frac{c-\mu}{\sigma}\right)$ (1)

$= 1 - \Phi\left(\frac{c-\mu}{\sigma}\right)$

(3)

hier: $\mu = 6$ $\sigma = \sqrt{16} = 4$

→ wat we willen: $1 - \Phi\left(\frac{c-6}{4}\right) \approx 0{,}025$

$\Phi\left(\frac{c-6}{4}\right) \approx 1 - 0{,}025 = 0{,}975$

→ in de tabel vinden we dat $\Phi(x) = 0{,}9750$ voor $x = 1{,}96$ (1)

→ $1{,}96 = \frac{c-6}{4}$ $c = 4 \cdot 1{,}96 + 6$

$c = 13{,}84$ (1)

2 a) (Check: moet ~~...~~ gevoelsmatig 1/2 zijn)

(4)

A = an random coin lands ~~op~~ on tail

$P(A) = \frac{4}{10} = 0{,}4$ (1½)

AB = lands on tail ~~...~~ and a ordinairy coin is selected

$P(AB) = \frac{2}{10} = 0{,}2$ (1½)

B = an ordinairy coin is selected.

$P(B) = \frac{2}{5} = 0{,}4$

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{0{,}2}{0{,}4} = \underline{\underline{1/2}}$ (1)

b) $P(C)$ = ~~...~~ prob ~~...~~ at least 1 is dubble headed

(3)

$= 1 - P(C^c) = 1 - \underbrace{P(X=0)}_{\text{none are dubble headed}}$ (1)

$P(X=0) = \frac{\binom{3}{3}}{\binom{5}{3}} = \frac{1}{\frac{5!}{3!\,2!}} = \frac{1}{10}$

dus $P(C) = 1 - \frac{1}{10} = \underline{\underline{\frac{9}{10}}}$ (2) = prob at least one is dubble headed.

(3)

3 Poison: $P(x=m) = e^{-\lambda} \frac{\lambda^m}{m!}$ with $\lambda = np = 2000 \cdot 0{,}0014 = 2{,}8$ (1)

(3)

(2)

$P(X=3) = e^{-2{,}8} \frac{2{,}8^3}{3!} = e^{-2{,}8} \cdot \frac{21{,}952}{6} \approx \underline{\underline{0{,}22}}$

4 total possible outcomes of the flips: $2^3 = 8$

(4)

$P(X=3) = 1/8$

$P(X=2) = 3/8$ (2½)

$P(X=1) = 3/8$

(giving the coins back is no (verlies??) lost / anti gain ?(? sorry forgot the word...)

$P(X^{0}=-1,-2,-3,-4,\ldots,-10) = 0$

$P(X=-8) = 1/8$

+ = 1 → klopt!

$E(X) = \sum_{i=-8}^{3} i\, P(X=i) = -8 \cdot 1/8 + 0 + 1 \cdot 3/8 + 2 \cdot 3/8 + 3 \cdot 1/8 = \underline{\underline{1/2}} = E(X)$ (1½)

5 a) $F(x) = \int_{-\infty}^{x} f(u)\,du$ (½)

(3)

voor $x < 2$: $F(x) = 0$ $\left(\int_{-\infty}^{2} f(u)\,du = 0\right)$

voor $2 \le x \le 4$: $F(x) = \int_{2}^{x} \frac{u}{6}\,du = \left[\frac{1}{12}u^2\right]_2^x = \frac{1}{12}u^2 - \underset{4/12}{1/3}$ (1) (½)

voor $x > 4$ $F(x) = 1$ $\left(\int_{-\infty}^{4} f(x)\,dx + \int_{4}^{x} f(x)\,du = 1 + 0 = 1\right)$

dus $F(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{12}u^2 - 1/3 & 2 \le x \le 4 \\ 1 & x > 4 \end{cases}$ (½) (½)

b) $E(X) = \int_{-\infty}^{\infty} x\, f(x)\,dx = \int_{2}^{4} x \frac{x}{6}\,dx = \frac{1}{18}\left[x^3\right]_2^4 = \frac{1}{18}(4^3 - 2^3)$

$= \frac{1}{18}(64-8)$

$= 56/18$

$= 28/9$ (1)

(3)

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{2}^{4} x^2 \frac{x}{6}\,dx = \frac{1}{24}\left[x^4\right]_2^4 = \frac{1}{24}(4^4 - 2^4)$

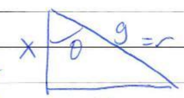
$= \frac{1}{24}(256-16)$

$= 10$ (1½)

$Var(X) = E(X^2) - E(X)^2$ (½)

$= 10 - \frac{28^2}{9^2} = 10 - \frac{784}{81} = \underline{\underline{\frac{26}{81}}}$

c) ~~(...)~~



y is given by $y^2 = 9^2 - x^2$

y is positive so $y = \sqrt{9^2 - x^2}$

$x = r\cos\theta$

$y = r\sin\theta$

$\theta = \cos^{-1}\left(\frac{x}{r}\right)$ → niet makkelijker

→ $E(Y) = \int_{-\infty}^{\infty} \sqrt{9^2 - x^2}\, f(x)\,dx = \int_{2}^{4} \sqrt{9^2-x^2}\, \frac{x}{6}\,dx$ (1) (½)

(nb $\frac{d(9^2-x^2)^{1{,}5}}{dx} = \frac{3}{2} \cdot -2x \cdot \sqrt{9^2-x^2} = -3x\sqrt{9^2-x^2}$)

$= \frac{-1}{18}\left[(9^2-x^2)^{1{,}5}\right]_2^4 = -\frac{(81-16)^{1{,}5}}{18} + \frac{(81-4)^{1{,}5}}{18} \approx \underline{\underline{1{,}87}}$ 8.42 (1) (½)

(202 / PTO)