Answers exam OBP 30 May 2023

holding costs

1a. $\min \sum_t \sum_n h_n s_{nt}$ ← inventory

s.t. $s_{nt} + x_{nt} - d_{nt} = s_{n,t+1} \quad \forall n, t$

production demand

$\sum_n u_{nm} x_{nt} \le r_{mt} \quad \forall t$

resource usage availability of resources

$x_{nt}, s_{nt} \geq 0 \quad \forall n, t$

1b. $n=1, m=1, T=1, u_{11}=1, s_{10}=0$.

$r_{11}=1, d_{11}=2$

you can produce 1 item but you need 2

1c. objective: add $+M \sum_m \sum_t v_{mt}$ M big

resource constraint: $\sum_n u_{nm} x_{nt} \le r_{mt} + v_{mt}$.

v_{mt} is the extra capacity needed for feasability, because of high costs will be avoided as much as possible

2a. $B(s,a) = \dfrac{\mathbb{P}(X > s-1) - \mathbb{P}(X > s)}{1 - \mathbb{P}(X > s)}$ with $X \sim \text{Poisson}(a)$

$s=5, a=4$: $\dfrac{0.371 - 0.215}{1 - 0.215} = 0.1987$

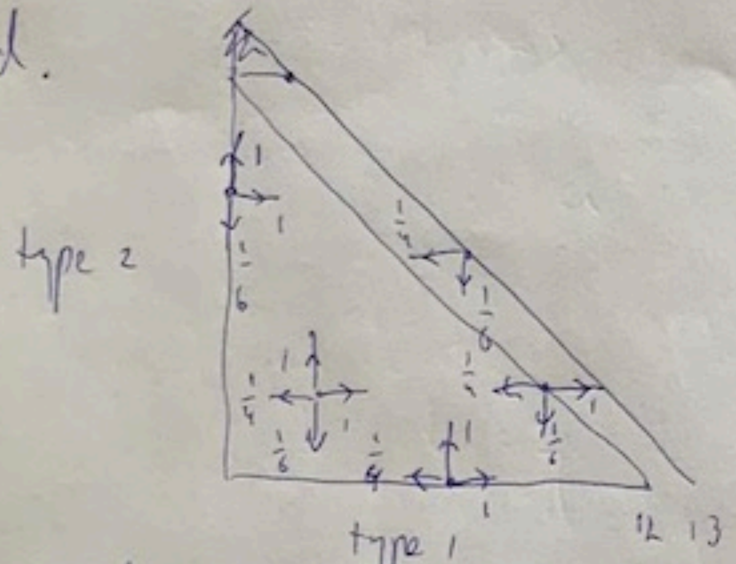
$s=8, a=6$: ≈ 0.12

b. $s=13, a=10$: ≈ 0.08

c. adv: not all nurses multi-skilled

disadv: lower occupancy

d.



2-dim because LOS are different

3a. $$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_{3,1} u_{t,1} + \dots + \beta_{3,6} u_{t,6} + \beta_{4,1} v_{t,1} + \dots + \beta_{4,51} v_{t,51} + \varepsilon_t$$

with $u_{t,i}$ and $v_{t,j}$ dummy variables such that

$u_{t,i} = 1$ iff day t is the ith day of the week and

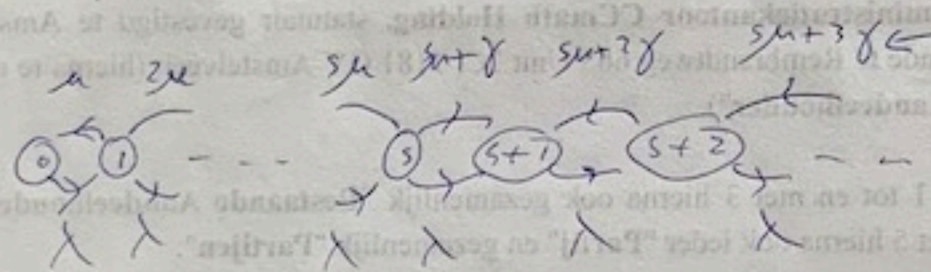
$v_{t,j} = 1$ iff day t is in week j; ε_t is noise.

3b. the estimation will always be ≥ 0 and seasonality is expected to have a multiplicative effect

3c. They often get connected thus you don't see their patience.

Kaplan-Meier: the patience of connected calls gets estimated by looking at later abandons.

3d.



3e. Because waiting time is not part of the state you cannot reproduce the routing

4a. $$V_t(x) = \max\Big\{\tfrac14 y_1 + \tfrac14 V_{t+1}(x-1) + \tfrac34 V_{t+1}(x),$$
$$\tfrac12 y_2 + \tfrac12 \quad '' \quad \tfrac12 \quad '' ,$$
$$\text{prices} \rightarrow \tfrac34 y_3 + \tfrac34 \quad '' \quad \tfrac14 \quad '' ,$$
$$y_4 + \quad '' \Big\}, \; x > 0$$

$$V_T(x) = 0, \; V_t(0) = 0.$$

b. $$V_{T-1}(1) = \max\left\{\tfrac14 4, \tfrac12 \cdot 3, \tfrac34 2, 1\right\} = \tfrac32 \text{ for prices } 2 \,\&\, 3.$$

$$V_{T-2}(1) = \max\left\{1 + \tfrac34 \cdot \tfrac32, \; \tfrac32 + \tfrac12 \cdot \tfrac32, \; \tfrac32 + \tfrac14 \cdot \tfrac32, \; 1\right\} =$$

$$\max\left\{\cancel{\tfrac74}\ \tfrac{17}{8}, \tfrac94, \tfrac{15}{8}, 1\right\} = \tfrac94, \text{ prices } 3.$$

c. overbooking: book more than capacity because some people will not show up

overselling: sell to high-paying customers even when no seats available (some people take other flight for a fee)

d. Make it possible to book more than capacity (eg $x < 0$) and compute $V_T(x)$ for $x < 0$, the costs of paying fees (fees times shortage of seats, for example using binomial distribution)

5.a. data science/AI is much more than running algorithms, requires other activities & skills

5b. paid capacity lost to activities that are not directly productive. 30%, 55% (5% margin)

5c. reinforcement learning