Exam Optimization of Business Processes
May 2023

This exam consists of **5** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal grade is 1, the maximal grade is 10. The points that can be achieved are indicated next to the questions.
The use of a calculator and a dictionary are allowed.
A table with the Poisson distribution is attached.

1a (0.5). Give the mathematical programming formulation of aggregate production planning and explain all variables.
b (0.5). Give a simple example for which the problem is infeasible. Give the value of all variables and explain why it is infeasible.
c (0.5). Extend the formulation to one that is always feasible but that gives the same solution as the original when that one is feasible. Make sure it is linear.

2. A hospital has two specialties with on average 1 arrival per day and average lengths-of-stay of 4 and 6 days, respectively.
a (0.5). Assuming different wards with 5 and 8 beds, compute the blocking probabilities per ward using the table with the Poisson distribution.
b (0.5). Do the same, but now when the wards are merged.
c (0.5). The hospital considers using earmarked beds. Give an advantage and a disadvantage of earmarking compared to merging.
d (0.5). The following policy is used for the merged ward: patients of type 1 are always admitted when a bed is free, patients of type 2 only when at least 2 beds are free. Give the state-transition diagram of the Markov process assuming exponential lengths-of-stay. Is it 1 or 2-dimensional?

3a (0.5). Formulate a linear regression model to predict daily arrivals in a call center with quadratic trend, intra-week seasonality and intra-year seasonality.
b (0.5). Give two reasons to apply these methods to the logarithms of the actuals instead of directly to the actuals.
c (0.5). Explain in what sense the patience of callers is censored and describe a method to "uncensor" the times that customers spent waiting.
d (0.5). Give the state-transition diagram of a queue that models a single-skill call center with callers with patience.
e (0.5). Routing in multi-skill call centers is usually based on the time spent in the queue of the longest-waiting customer. Explain why this makes formulating a Markov process of the call center difficult.

4. A hotel uses 4 prices for a certain night. Every day there is always 1 interested customer, with probability of willingness-to-pay for each one of the 4 prices equal to 1/4. There is buy-down, thus if you use the lowest price then you sell with certainty a room for that price, if you use the 2nd lowest price then the probability of sales is 3/4, etc.
a (0.5). Formulate the dynamic programming recursion for this problem (for every day and every number of available rooms).
b (0.5). Use this formulation to compute the total expected revenue if there are two days and 1 room left for prices of 4, 3, 2 and 1.
c (0.5). Give the definitions (according to the slides) of overbooking and overselling.
d (0.5). Explain how you could extend the dynamic programming formulation to include overbooking.

5a (0.33). What does Haagoort (Intellerts) mean by "AI is a red herring"?
5b (0.33). Give the definition of Bakker (CCmath) of shrinkage. How high is it in large firms and government?
5c (0.33). Which AI technology does Duijndam (Air France KLM) use to maximize revenue on paid seats?

Table with values of $\mathbb{P}(X > k)$ with X a Poisson distributed random variable with mean μ

	values of μ									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000