Answers exam OBP May 2022

1. x_i = starttime (decision var), s_i = duration, d_i = due date
$i = 1, \dots, n$.

a. flow time $= \sum_{i=1}^{n} (x_i + s_i)$

makespan $= \max_i (x_i + s_i)$

tardiness (of job i) $= (x_i + s_i - d_i)^+$,

total tardiness $= \sum_i (x_i + s_i - d_i)^+$.

b. $\min z$

s.t. $z \geq x_i + s_i \quad \forall i$

$x_i + s_i \leq x_j + M y_{ji} \quad \forall i \neq j$, M big

$y_{ij} + y_{ji} = 1 \ \forall i \neq j$, $y_{ij} \in \{0,1\}$

$x_i \geq r_i$ (or 0) $\forall i$.

c. $\min \sum_{i=1}^{n} e_i$

s.t. $e_i \geq x_i + s_i - d_i \quad \forall i$

$e_i \geq d_i - x_i - s_i \quad \forall i$

plus other inequalities.

2a. 110, 101, 011

$\phi(y_1, y_2, y_3) = 1 - (1-y_1y_2)(1-y_1y_3)(1-y_2y_3) =$

$\ldots = y_1y_2 + y_1y_3 + y_2y_3 - 2y_1y_2y_3$ (using $y^2 = y$ if $y \in \{0,1\}$)

$\Phi(p_1, p_2, p_3) = p_1p_2 + p_1p_3 + p_2p_3 - 2p_1p_2p_3$

b. 1 component, $X \sim N(3, 0.5)$:

$\mathbb{P}(X > 4) = \mathbb{P}\left(\frac{X-3}{0.5} > \frac{1}{0.5}\right) = \mathbb{P}(Y > 2)$ with $Y \sim N(0,1)$

$= 0.5 - 0.477 = \cancel{0.977}\ 0.023$

$\Phi(p_1, p_2, p_3) = 3(0.023)^2 - 2\,(0.023)^3 = 0.0015.$

$\mathbb{P}(\text{system down}) = 1 - 0.0015 = 0.9985.$

c. Erlang B — maintenance problem

arrivals — repairs

departures — failures

$B(s, a) = \mathbb{P}(\text{all up}) = \frac{a^2/2!}{1 + \frac{a}{1!} + \frac{a^2}{2!}} = \frac{\left(\frac{3}{2}\right)^2/2}{1 + \frac{3}{2} + \frac{9}{8}} = \frac{\frac{9}{8}}{\frac{8+12+9}{8}} =$

$s = 2$, $a = \lambda\beta = \frac{1}{2}\,3 = \frac{3}{2}$

$\frac{9}{29} \approx 0.31.$

d. Exact, Erlang B is insensitive to service time dist = failure dist.

3a. x_i realizations, $\hat{x}_i$ forecasts

$$\text{WAPE} = \frac{\sum_i |x_i - \hat{x}_i|}{\sum_i x_i}.$$

b. Not prone to outliers

Explainable

Linear in "real-time mgmt" costs

c. The forecast is at best the arrival rate: thus there is always a "Poisson error".

$$\text{APE} = \frac{\mathbb{E}|N - \mathbb{E}N|}{\mathbb{E}N} \approx \sqrt{\frac{2}{\lambda\pi}} \text{ for } N \sim \text{Poisson}(\lambda).$$

d. Redials occur more often when SL is bad, if they are not taken into account they are an additional factor for errors. This makes the WAPE bigger. Redials can be excluded by calculating the "fresh" arrivals.

4a. $N \sim \text{Poisson}(1)$: $\mathbb{P}(N=0) = 0.37$, $\mathbb{P}(N=1) = 0.37$

$\mathbb{P}(N \geq 2) = 0.26$

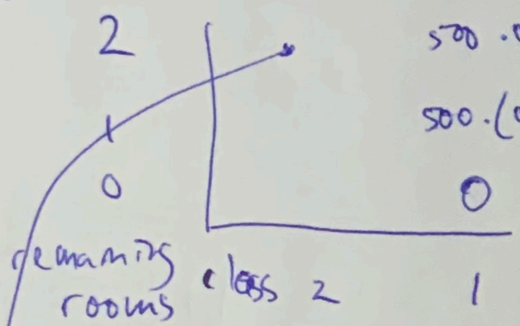
$$s^* = \max_s \{ y_1 \mathbb{P}(D_1 \geq s) > y_2 \}$$

$s=0$: $500 > 200$ ✓

$s=1$: $500 \cdot 0.63 > 200$ ✓ $\longleftarrow$ $s^* = 1$.

$s=2$: $500 \cdot 0.26 < 200$

b.



$500 \cdot 0.37 + 1000 \cdot 0.26 = 445$

$500 \cdot (0.63) = 315$

$$V_2(2) = \max \{ \underset{\substack{\uparrow \\ \text{reserve } 0}}{400}, \; 200 + \underset{\substack{\uparrow \\ 1}}{315}, \; 0 + \underset{\substack{\uparrow \\ 2}}{445} \}$$

515 with optimal action reserve 1.

c.

d. Now

$$s^* = \max_s \{ y_1 \mathbb{P}(D_1 \geq s) > y_2 + (y_1 - y_2 - \underset{\substack{\downarrow \\ \text{refund } 50}}{z}) \mathbb{P}(D_1 \geq s) \}$$

$$= \max_s \{ (y_2 + z) \mathbb{P}(D_1 \geq s) > y_2 \}$$

$s=0$: $250 > 200$ ✓ $\longleftarrow$ $s^* = 0$, no reservation.

$s=1$: $250 \cdot 0.63 < 200$ ✗

5a. prescriptive analytics

b. the ML algorithm

c. leisure customers stay at least a weekend, business travelers do not.