

Exam Optimization of Business Processes

31 May 2022

This exam consists of **5** problems, each consisting of several questions.

All answers should be motivated, including calculations, formulas used, etc.

It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.

The minimal grade is 1, the maximal grade is 10. The points that can be achieved are indicated next to the questions.

The use of a calculator and a dictionary are allowed.

A table with the normal distribution and a table with the Poisson distribution are attached.

1. Consider single-machine scheduling. Introduce notation for the start times, durations and due dates of jobs.

a (0.5). Give formulas for the flow time, makespan, and tardiness.

b (1). Formulate a linear model that minimizes the makespan for a single-machine scheduling problem with n jobs.

c (0.5). To avoid having to store large quantities of finished products the objective is now to minimize the sum of absolute differences between finish time and due date. Reformulate the linear model and make sure it is again linear.

2. Consider a 2-out-of-3 system (with warm stand-by).

a (1). Give the minimal path sets and use that to determine $\Phi(p_1, p_2, p_3)$.

b (0.5). The time to failure of all components is approximately normally distributed with mean 3 and standard deviation 0.5. Calculate the probability that the system is down at time 4.

Now there is a single repairman with exponentially distributed repair times with average 2.

c (1). Use the Erlang B model to determine the long-run probability that all components are up.

d (0.5). Is the answer to c an approximation or exact? Motivate your answer.

3. We consider the forecasting of call arrivals in a call center.
- a (0.25). Give a formula for the WAPE.
 - b (0.25). Give two reasons why the WAPE is a good choice for measuring forecasting error.
 - c (0.25). Explain why the forecasting error can hardly ever be 0 and give a formula for the minimal expected APE.
 - d (0.25). Explain how redials interfere with forecasting, the effect on the forecasting error, and how this effect can be avoided.

4. A small hotel has only 2 rooms and 2 possible prices: 500 and 200. 200 is used until 1 month before the booking date, 500 after that. The demand for the high-paying class is Poisson with expectation 1, the demand for price 200 exceeds 2.
- a (0.5). Determine the number of rooms to reserve for the high-paying class using a newsvendor-type method.
 - b (0.5). Do the same, but now using dynamic programming.
 - c (0.5). What is expected revenue under the optimal policy?
 - d (0.5). Now it is possible for the hotel to cancel reservations for a refund of 50. What is now the optimal policy?

- 5a (0.33). According to Gertjan de Lange of AIMMS, which part of Analytics is least used and has the highest added value?
- b (0.33). What is according to Martin Haagoorts of Intellerts, the **smallest** IT challenge in data science?
- c (0.33). According to Daniel Hopman of MAF, what is the most common way for airlines to segment customers?

Table with values of $\mathbb{P}(X > k)$ with X a Poisson distributed random variable with mean μ

	values of μ									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table with values of $\mathbb{P}(0 < X < x + y)$ with X a random variable with a standard normal distribution

	values of y									
values of x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499