Exam Optimization of Business Processes
29 June 2021

This exam consists of **5** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal grade is 1, the maximal grade is 10. All questions (1a, 1b, etc.) give the same number of points (0.5).
The use of a calculator and a dictionary are allowed.
A table with the normal distribution and a table with the Poisson distribution are attached.

1. Consider a 2-out-of-3 system (with warm standby).
a. Determine the minimal path sets and formulate ϕ and Φ.
Let X be the lifetime of the system. Assume all 3 components have exponential lifetimes with the same average.
b. Use the answer to a) to find a formula for $\mathbb{P}(X > t)$.
c. Use the formula $\mathbb{E}X = \int_0^\infty \mathbb{P}(X > t)dt$ and the result found under b) to compute $\mathbb{E}X$.
d. Compute $\mathbb{E}X$ directly using the memoryless property of the exponential distribution.
e. Under which circumstances does a component have an exponential lifetime? Explain your answer.

2. A ward in a hospital has on average 2 arrivals per day and an expected length of stay of 4 days.
a. Give two reasons why the Erlang B model does not exactly fit reality in most hospitals, and explain why it is ok to use it.
b. Compute the blocking probability using Erlang B for 8, 9 and 10 beds.
c. Compute the differences of the numbers found under b). Are they increasing or decreasing? Explain your findings.
d. For the same arrival rate and service times, and 10 servers, compute the delay probability of the Erlang C model using the formula $C(s, a) = sB(s, a)/(s - a(1 - B(s, a)))$ and use this to compute $\mathbb{P}(W_Q > 2 \text{ days})$.

3a. Formulate an integer linear program that minimizes staffing costs in a single-skill inbound call center subject to the constraint that the service level is attained in every interval, assuming that the service level is computed by a model such as Erlang C.
3b. Change this problem you just formulated into one where only the weighted average service level over the whole has to be met. Make sure the formulation remains linear.
3c. Now we add emails to the problem of a), in any interval an agent can do either inbound or email. Emails have to be answered the same day, and of course they cannot be answered before they arrive. Extend the formulation of a) such that the optimal combination of shifts and handling of email is determined.

4. Consider a revenue management model with 2 classes, class 2 books before class 1 (the Littlewood model). The revenues are: $y_1 = 4$, $y_2 = 1$. The demand of class 1 and 2 are Poisson distributed with average 10, overall capacity is 20.
a. What is the optimal amount of capacity that needs to be reserved for class 1?
b. Write down a formula to compute the expected revenue for a given reservation level.
c. It is now possible to use overselling. When a ticket is bought back from a class-2 customer then this costs 1 more. What is the optimal reservation level?
d. For this situation, write down a formula to compute the number of unsold seats for a given reservation level.

5a. Which technique did Alex Roubos show to get smooth intra-day patterns in call centers?
5b. What is, according to Rik van Leeuwen, the often-used definition of revenue management that gives 3.5M hits on Google? "Selling the ..."

Table with values of $\mathbb{P}(X > k)$ with X a Poisson distributed random variable with mean μ

	values of μ									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table with values of $\mathbb{P}(0 < X < x+y)$ with X a random variable with a standard normal distribution

	values of y									
values of x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499