Answers Exam OBP 25 May 2021

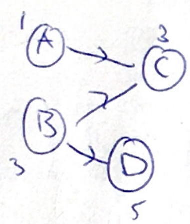
1a

min duration = 6
critical path = B − C
slacks: A: 2, B, C: 0, D: 1.

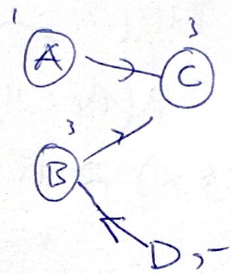
b.

$\exp = \frac{2}{3} \cdot 5 + \frac{1}{3} \cdot 5.5 = 5.166$

c.



or



duration = 8

shorter: first B then D

duration = 11

d

z = project duration

s_x = start time of x

y = 1 if B → D, 0 if D → B

min z

s.t. $s_x \geq 0$; $s_x + d_x \leq z$ (← durations)

$s_A + d_A \leq s_C$; $s_B + d_B \leq s_C$ (A→C, B→C)

$s_B + d_B \leq s_D + (1-y) M$ → big

$s_D + d_D \leq s_B + y M$

$y \in \{0, 1\}$

2a. $1-\Phi\left(\frac{8-7}{\sqrt{\frac{7}{4}}}\right) = 1-\Phi(0.75) = 1-\left(\frac{1}{2}+0.273\right) = 0.227$

2b. $\int_{8}^{\infty} x \frac{1}{\sqrt{2\pi\frac{7}{4}}} e^{-\frac{1}{2}\left(\frac{x-7}{\sqrt{\frac{7}{4}}}\right)^2} dx$

2c. Separate is better. overall variability is more reduced in that case.

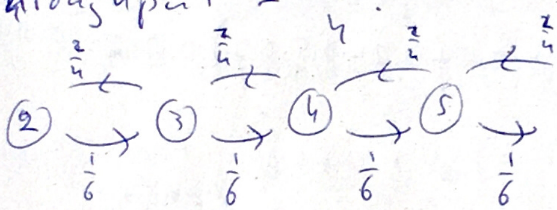
3a. $1-SL = \mathbb{P}(\text{delay})\, e^{-(s\mu-\lambda)t} = \frac{4}{6} e^{-\left(\frac{1}{4}-\frac{1}{6}\right)} \approx 0.61$

↳ $s=1$: ρ

$\Rightarrow SL \approx 39\%$

throughput = $\frac{1}{4}$.

3b.



convolution as Erlang C or argument as above:

$1-SL = 1\cdot e^{-\left(2\frac{1}{4}-\frac{1}{6}\right)} \approx 0.71 \Rightarrow SL \approx 29\%$.

rho on inbound =1/6*4 = 2/3, thus on av 4/3 server is avaiable for outbound, throughput = 4/3*1/4=1/3

3c. only schedule outbound when both are available.

4a.

$$V_t(x) = \min_{a \in \{\max\{0,\, d_t - x\}, \dots, M - x + d_t\}} \{hx + K I\{a>0\} + V_{t+1}(x - d_t + a)\}$$

b.

$$V_2(0) = \min_{1 \le a \le 3} \{1 \cdot 0 + 10 I\{a>0\}\} = 10 \qquad a^* \in \{1, 3\}$$

$$V_2(1) = \min_{0 \le a \le 2} \{1 + 10 I\{a>0\}\} = 1 \quad (a^* = 0)$$

$$V_2(2) = \min_{0 \le a \le 1} \{2 + 10 I\{a>0\}\} = 2 \quad (a^* = 0)$$

$$V_1(0) = \min_{2 \le a \le 4} \sout{\{0 + 10 + V_2(0)\} = 20 \ (a^* = 2)}$$

$$\{0 + 10 + V_2(a-2)\} = 11 \quad (a^* = 3)$$

$$V_1(1) = \min_{1 \le a \le 3} \{1 + 10 + V_2(a-1)\} = 12 \quad (a^* = 2)$$

$$V_2(2) = \min_{0 \le a \le 2} \{2 + 10 I\{a>0\} + V_2(a)\} = 12 \quad (a^* = 0)$$

order all at once and delay ordering when possible.

c. end of day: make sure you have enough inventory, i.e.,

$$a \in \{\max\{0,\, d_t + d_{t+1} - x\}, \dots, M - x + d_t\}.$$

end of next day: extra state variable for order in from t

$$V_t(x, y) = \min_a \{hx + K I\{a>0\} + V_{t+1}(x + y - d_t, a)\}.$$

$$a \in \{\max\{0,\, d_t + d_{t+1} - x - y\}, \dots, M - x - y + d_t\}$$

5a. weekend stay = leisure

5b. (smoothing) splines

5c. linear programming / optimization