Exam Optimization of Business Processes
29 May 2018

This exam consists of **5** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal grade is 1. All questions (1a, 1b, etc.) give the same number of points.
The use of a calculator and a dictionary are allowed.
A table with the normal distribution and a table with the Poisson distribution are attached.

1. Consider a tandem system with 2 queues, Poisson arrivals with rate 1, and expected service time 0.9 at both stations, infinite queues.
a. Give the expected sojourn time (waiting times plus service times) if the service times are exponential.
b. Give the expected sojourn time if the service times at queue 1 are exponential and at queue 2 deterministic.
c. Give an approximation of the expected sojourn time if the service times at queue 1 are deterministic and at queue 2 exponential.
d. Explain why the results under b and c differ and relate this to the improvement method called lean.

2. A system consists of 3 components A, B and C. A and B are in parallel (in warm stand-by), C is placed in series. Thus A or B has to be up, and C always has to be up for the system to be working.
a. Give ϕ and Φ.
b. A has a uniform [0,2] distribution, B is exponential with average 3, C is lognormal, its log has mean 0.5 and variance 1. Determine the probability that the system is up at time 1.
c. Now B is in cold stand-by. Answer the same question.

3. Two hospital wards have a load of 10 and 12 beds, and equal ALOS (average length of stay).
a. Compute the rejection probability using the Erlang B formula.
b. Approximate the rejection probability using the normal distribution.
c. Now the wards are merged. Approximate the rejection probability using the normal distribution.
d. Now every ward has 6 earmarked beds and 12 beds are shared. Will the rejection probability be higher or lower than the average of the answers between b and c? Motivate your answer.
e. Mention 2 methods by which you can calculate this number.

4. A service center is operated during a day. This day is split into T intervals of equal length. For every interval t the optimal number of employees is determined, s_t. Employees work according to shifts. There are K different shifts.
a. Formulate a linear programming problem that assigns employees to shifts such that the sum of absolute deviations from the optimal numbers s_t are minimized.
b. Change the formulation such that the number of intervals, for which the number scheduled is unequal to s_t, is minimized. Make sure it remains linear.
c. How can you determine if it is possible to find a schedule that meets exactly s_t in every interval?
d. For general LP problems, formulate a general method to avoid that a problem might be infeasible.

5. The inventory of a certain item is controlled over T time units. Inventory can be between 0 and M, demand is d every time unit. Initial stock is 0, the fixed order costs are K. Replenishments occur at the beginning of the day and are immediate.
a. Formulate a dynamic programming recursion that minimizes the total costs.
Now demand is stochastic, with $p(d)$ the probability that the demand is d. Backorders cost q per item.
b. Formulate a dynamic programming recursion that minimizes the total expected costs.
Demand that occurs when inventory is 0 is now lost. Lost sales cost r per item.
c. Formulate a dynamic programming recursion for this problem.