Exam Optimization of Business Processes
24 May 2016

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give the same number of points.
The use of a calculator and a dictionary are allowed. A table with the normal distribution is attached.

1. A production process consists of two steps. The processing times for each step are independent and uniform distributed on $[0, 1]$. Orders arrive according to a Poisson process with rate 1.
a. The first configuration allows for an infinite buffer between the processing steps. Is the flow between station 1 and 2 Poisson? Motivate your answer.
b. Give a reasonable approximation of the total expected time between order arrival and order completion.
Now the process is changed as follows. Both steps are done in parallel. A new order is taken into production when both steps of the previous order are done.
c. Compute the first and second moment of the time it takes to process one order. (Hint: compute first the distribution function of this time.)
d. Compute the expected time between order arrival and order completion.

2. A project consists of 6 activities, numbered $1, \ldots, 6$. Their respective durations are 2, 2, 3, 1, 4, and 2. Activity 1 needs to be done before 2 and 5 start, 2 before 3 and 6 start, 4 before 5 starts, 5 before 6 starts.
a. Determine the earliest finish time and latest start time of each activity.
b. Assume durations are random with expectation as given and each with standard deviation of 1. Use a normal approximation of the duration of the critical path to estimate the probability that the project takes longer than 9 time units. A table with the normal distribution is attached. (Hint: $\sqrt{3} \approx 1.73$.)
c. Give two reasons why reality might deviate considerably from this approximation.

3a. Formulate the call center shift scheduling problem that requires the service level to be attained in every interval.
3b. Give two reasons why this leads to overstaffing.
3c. To avoid overstaffing the constraint on each interval is replaced by a second component of the objective that counts the total difference over all intervals between required staffing and planned staffing. Formulate the new problem as an integer linear program.
3d. At the beginning of the day there is a certain number of emails that needs to be done before the end of the day. Add this constraint to the formulation.

4. A hotel receives individual guests and couples. Single travelers pays 100 Euros, couples 120. There are 20 rooms, each room is suitable for both types of bookings. Consider the bookings for a single night. Assume that demand is discretized such that there is at most 1 booking per period.
a. Formulate the dynamic programming equation that maximizes the expected revenue.
b. Do 4 iterations with 2 room and booking probability 0.4 for each type (thus with probability 0.4 single traveler, 0.4 couple, 0.2 no demand).
Now there are single-bed rooms and double-bed rooms. Single travelers can use a 2-person room but not vice versa.
c. Formulate an appropriate state space for this problem.
d. Write down the dynamic programming equations. Think about all boundary cases ($t = 0$, no 1-person rooms left, etc.).

Table with values of $\mathbb{P}(0 < X < x+y)$ with X a random variable with a standard normal distribution

	values of y									
values of x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499