Exam Optimization of Business Processes
26 May 2015

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give the same number of points.
The use of a calculator and a dictionary are allowed.

1. Consider an $M|G|1$ queue with mean interarrival time 1 and a service time that can be 0, 1, or 2, with probability 0.5, 0.25, and 0.25, respectively.
a. Compute the first and second moments of the service time distribution.
b. Compute the expected waiting time for the FCFS queueing discipline.
c. Compute the expected waiting time if we give (non-preemptive) priority to customers with a short waiting time.
We change the policy as follows: we interrupt service if a customer with a shorter service time arrives. If we resume with an interrupted customer then we start from the beginning with its service.
d. What do you expect the consequences to be for the waiting times of the different customer classes? Motivate your answer.
e. Do you think that the system is stable? Motivate your answer.

2. Consider a 2-out-of-3 system (with warm standby).
a. Determine the minimal path sets and formulate ϕ and Φ.
Let X be the lifetime of the system. Assume all 3 components have exponential lifetimes with the same average.
b. Use the formula $\mathbb{E}X = \int_0^\infty \mathbb{P}(X > t)dt$ and the result found under a) to compute $\mathbb{E}X$.
c. Compute $\mathbb{E}X$ directly using the memoryless property of the exponential distribution.
d. Under which circumstances does a component have an exponential lifetime? Explain your answer.

3. A hospital has two wards having about the same average lengths of stay. The hospital considers merging both wards.
a. Give the birth-death process of the merged ward for the situation where all patients are admitted unless there is no capacity. Give the rejection probability for both classes.
b. Give a relation between the rejection probabilities before merging and after merging. What can we say about the individual classes?
c. Formulate two disadvantages of the merged system.
To tackle the disadvantages of the merged ward the hospital considers two solutions: having dedicated beds for 1 or 2 classes or having an admission policy in which one of the classes is rejected if there are few beds left.
d. Draw the state-transition diagram of both policies and relate the systems to the answers given under c).

4. Consider a revenue management problem with capacity C, 2 classes with prices $y_1 > y_2$, a period T in which bookings can take place, and non-homogeneous Poisson demand with rate $\lambda_t(i)$ for time t and class i. Customers book in their class even if cheaper tickets are available (no diversion).
a. Describe 2 situations in which it might be realistic to assume no diversion.
b. Formulate a discrete-time dynamic programming approximation for this problem. Explain how you discretized the customer arrival process.
c. Prove that when it is optimal to accept class 2 it is also optimal to accept class 1.
d. Formulate a dynamic programming approximation for the case with diversion: when class 2 is available class-1 customers book class 2.
e. For the problem with diversion, add the following restriction to the policy: once class 1 is closed it will not open again. Formulate a dynamic program for this situation.