Exam Optimization of Business Processes
8 July 2013

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give the same number of points.
The use of a calculator and a dictionary are allowed.
A table with the normal distribution and a table with the Poisson distribution are attached.

1a. Compute $\mathbb{E}S$ and $\mathbb{E}S^2$ for $S = ZX + (1-Z)Y$ with X, Y and Z independent, $Z \in \{0,1\}$, $X \sim \exp(\mu_1)$ and $Y \sim \exp(\mu_2)$. (Note that S has a so-called hyperexponential distribution.)
1b. Compute the expected waiting for an M/G/1 queue with arrival rate $\lambda = 1$ and service time distribution S with $\mathbb{P}(Z=0) = 0.5$, $\mu_1 = 1$ and $\mu_2 = 2$.
1c. Compute the expected waiting for both priority policies of a non-preemptive priority queue with two classes with each arrival rate 0.5 and exponential service times with rates 1 and 2.

2a. A system has 4 minimal path sets: $\{A,B,C\}$,$\{D,B,C\}$, $\{A,E\}$, $\{D,E\}$. Formulate ϕ and Φ.
b. Each component has a lifetime that has a lognormal distribution where the underlying normal distribution has average -3 and variance 4. What is the probability that an arbitrary component is down at time 1?
c. Compute the probability that the system is down at 1 assuming all component have independent lifetimes as described under b.

3a. Give the mathematical programming formulation of aggregate production planning.
3b. Extend this formulation to allow for backorders of at maximum 1 time unit. There are costs for every unit that are backordered.
3c. Give an example of a problem (i.e., give the values) for which backordering is optimal but not necessary. Thus give a feasible suboptimal solution that uses no backorders, give a better solution that uses backorders and argue why it is optimal.

4a. A hotel has 20 identical rooms and 2 types of offers, with prices 100 and 150 Euro per night. Low-price customers book before high-price customers. Demand for a particular night is Poisson with mean 10 for each class. Determine the optimal numbers of rooms to reserve for high-price customers.
4b. Give the solution to the newsvendor problem in terms of the inverse to the demand function and explain the relation with problem 4a.
4c. Now we split up the 150 Euro-offer in 2 offers such that there is a Poisson(5) demand for price 150 and during the last days a Poisson(5) for price 180. Determine the protection levels according to the EMSR-a and EMSR-b algorithms.
4d. Explain a method to find the optimal protection levels.