Exam Optimization of Business Processes
28 May 2013

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give the same number of points.
The use of a calculator and a dictionary are allowed.
A table with the Poisson distribution is attached.

1a. Consider an M/M/1 queue with FCFS service discipline $\lambda = 0.8$ and $\mathbb{E}S = 1$. Give the expected waiting time in the queue $\mathbb{E}W_q$.
b. We change the service order in the following way: we use a 2-class non-preemptive priority policy, where class 1 are all customers with $S < t$ for some $t > 0$. Give the expected waiting time for both classes and the overall expected waiting time.
c. Prove that the answer found under b is smaller than the one found under a.
d. Would a preemptive priority policy give different results? Motivate your answer.

2a. A system consists of 2 components in series. Each component has one spare component. Formulate the minimal path sets, ϕ, and Φ. (The spare components are in warm standby.)
b. Derive the distribution if the lifetime of the system of all component have an exponential lifetime with average 1.
c. Now one of the spare components is multi-functional, it can replace both components. Formulate again the minimal path sets, ϕ, and Φ.

3a. A hospital ward has an average demand of 3, an average length of stay of 2, and 6 beds. What is the rejection probability? And the occupancy of the beds?
b. The hospital works with an "overflow" ward: when the specialized ward is full then the patients go to a joint ward where there are always beds available. What is the average number of beds occupied at this ward by the patients of question a?
c. The manager of the overflow ward complains that the arrivals are very "peaked": for a long time there are no arrivals and then a few shortly after each other. Can you explain this?
d. What is the advantage of having a "joint" overflow ward shared between different specialties? Please name also 3 disadvantages of such a joint overflow ward.

4a. Consider a call center with 1 type of inbound calls and staffing requirements for each interval. Consider the problem of assigning agents to shifts as to minimize total costs, where every shift has a possibly different cost. Formulate a mixed-integer programming (MIP) formulation of this problem.
b. Consider now a call center with 2 types of calls and single- and multi-skilled agents. All agents always work on 1 skill but multi-skilled agents can change skill from interval to interval. Costs depend on shift and skills. Formulate again an MIP that minimizes the costs.
c. Now we have 1 type of inbound calls and 1 type of emails, single- and multi-channel agents, agents work again always on 1 channel but multi-channel agents can change channel. Email can be handled in the interval in which they arrive or in the interval right after it. Costs depend on shift and channels. Formulate again an MIP that minimizes the costs.

Table with values of $\mathbb{P}(X > k)$ with X a Poisson distributed random variable with mean μ

	values of μ									
values of k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000