Exam Optimization of Business Processes
27 August 2012

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions give the same number of points.
The use of a calculator and a dictionary are allowed.

1a. Calculate $B(s,a)$, the blocking probability in an Erlang B system, with $\lambda = 2$, $\beta = 1$ and $s = 3$.
b. A well-known formula relates $B(s,a)$ to $C(s,a)$, the delay probability in the Erlang C system with the same parameters: $C(s,a) = sB(s,a)/(s - a(1 - B(s,a)))$. Use this to compute the service level $\mathbb{P}(W_Q \leq 1/3)$.
c. Prove and explain intuitively why $C(s,a) > B(s,a)$ if $a > 0$.
d. Which service discipline is used in the Erlang C model? Suppose it is changed to LCFS (last-come-first-served). Make an educated guess what this would mean for the service level and explain why.

2. Consider a series of 2 subsystems which each consists of 2 parallel components (with warm stand-by).
a. Calculate ϕ and Φ.
b. Compute the probability that the system is up at 1 when the lifetimes are exponentially distributed with average 1.
c. Answer the same question when the spare components are in cold standby.

3a. A project has the following activities:

Activity	Preceding activities	Duration
A	-	1
B	E	1
C	E,F	2
D	B,C,G	1
E	A,F	2
F	-	2
G	-	3

Assume for the moment that there are enough resources.
a. Make a graph representation of this project.
b. Compute the earliest finish time of the project and all earliest and latest starting times of the activities.
c. Give the definitions of slack, critical activity, and critical path.
d. Compute in the example project the slack of each activity. What is the critical path?
Suppose that activities E and G use the same resource. Therefore they cannot be scheduled at the same time.
e. What is now the earliest finish time of the project?
f. Prove that the solution to d. gives indeed the earliest finish time possible.

4. Consider 2 $M|D|1$ queues with load 80% and service times of 1 and 10 minutes, respectively.
a. Calculate the expected waiting time in both queues, and for all customers together.
The system is changed. Both arrival steams now arrive in a single queue with a server that is twice as fast: service times are now 0.5 and 5 minutes.
b. Calculate the expected waiting time for both customer types (under FCFS). Explain intuitively your findings.
c. Does a nonpreemptive priority rule change this? Give the expected waiting times under both policies.
d. Would a preemptive priority rule give different results? Motivate your answer.