

Exam Optimization of Business Processes

23 May 2011

This exam consists of 4 problems, each consisting of several questions.

All answers should be motivated, including calculations, formulas used, etc.

It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.

The minimal note is 1. All questions count equally.

The use of a calculator and a dictionary are allowed.

A table with the Poisson distribution is attached.

1. Consider a model for the waiting times of walk-ins for a CT scanner. Scan times are approximately normally distributed. There are two types of patients, one with average scan time of 15 minutes and standard deviation 5 minutes, the other with average 25 and standard deviation 10. The arrival processes are approximated by Poisson processes with averages 2 and 1 per hour.

- a. What is the load to this system?
- b. Calculate the weighted average scan time and standard deviation.
- c. Calculate the expected waiting time for the FCFS discipline and for both non-preemptive priority orders.

2. Consider a system with 4 components. All components have an exponentially distributed lifetime with expectation 1. Two components need to be up to have a functioning system. The system starts with components 3 and 4 as spare parts.

- a. Calculate the expected time that the system is up when components 3 and 4 are in cold stand-by.
- b. Calculate the expected time that the system is up when components 3 and 4 are in warm stand-by.
- c. Calculate ϕ and Φ for this system.
- d. What is the probability that the system is up after 2 time units?

3. A call center planner uses the Erlang C formula for computing the service level.
- a. Give 3 aspects in which the Erlang system does not model most call centers exactly, and explain how this influences the service level.

The planner estimates the input parameters as follows: $\lambda = 10$ and $\beta = 2$. With 24 agents the probability of waiting less than 20 seconds is 0.85, according to the Erlang C formula.

- b. What is the productivity?

A colleague analyses the data and says that λ is not always exactly 10, but that it can be somewhere between 9 and 11.

- c. How many agents would you schedule to be sure to have approximately an 80% service level? What can you say about the productivity?
- d. Explain two possible measures in many call centers that can help to deal with a λ that is not completely known, such that both the service level and the productivity are high.

4. Consider a revenue management problem with two classes of customers, type 2 books before type 1. Type 1 products cost 10, type 2 products cost 6. Total capacity is 16, and the demand for class 1 is Poisson distributed with expectation 10, the demand for class 2 can be assumed to be ∞ . Type 2 purchases can be cancelled by paying a fine of 2. Type 2 customers are only cancelled where there is demand of type 1 without capacity. Consider the following simple reservation policy: as long as there is capacity type 2 bookings are made, and for every type 1 reservation a type-2 booking is cancelled.

- a. Estimate the expected total revenue.

Now a booking limit for type-2 customers is used (while type-2 bookings can still be cancelled for a fine of 2).

- b. Calculate the optimal booking limit.
- c. Estimate again the expected total revenue.

Table with value of $P(X > k)$ with X a Poisson distributed random variable with mean μ

[illegible]