# Exam Optimization of Business Processes
# 31 August 2010

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions count equally.
The use of a calculator and a dictionary are allowed.
A table with the Poisson distribution is attached.

1. Consider a revenue management model with 2 classes, the Littlewood model. The revenues are: $y_1 = 3$, $y_2 = 1$. The demand of class 1 is Poisson distributed with average 8.
a. What is the optimal amount of capacity that needs to be reserved for class 1?
It is now possible to use overselling. When a ticket is bought back from a class-2 customer then this costs 1 more.
b. What is the optimal reservation level?
c. Describe a method to compute the expected number of unused capacity for a given reservation level, assuming ample demand of class 2.

2a. Formulate the call center shift scheduling model with $K$ different shifts and a service level constraint for each time interval.
b. Extend the shift scheduling model to allow for an overall service level constraint.
c. Reformulate if necessary the answer to b to make the model linear.

3. A project has the following activities:

| Activity | Preceding activities | Duration |
|---|---|---|
| A | - | 2 |
| B | A | 3 |
| C | A | 2 |
| D | C | 1 |
| E | B,D,G | 2 |
| F | - | 3 |
| G | C,F | 2 |

Assume for the moment that there are enough resources.
a. Make a graph representation of this project.
b. Compute the earliest finish time of the project and all earliest and latest starting times of the activities. (Hint: renumber first the activities.)
c. Give the definitions of slack, critical activity, and critical path.
d. Compute in the example project the slack of each activity. What is the critical path?
Suppose that activities B and C use the same resource. Therefore they cannot be scheduled at the same time.
e. What is now the earliest finish time of the project?
f. Prove that the solution to d. gives indeed the earliest finish time possible.

4. The waiting line for an MRI facility is modeled as an M/M/1 queue, with the additional feature that the arrival rate decreases linearly in the number of waiting patients until it reaches 0.
a. Model this system as a birth-death process.
b. Formulate the stationary probabilities of this process.
c. What is the most likely state in this system, i.e., the state with the highest stationary probability?
d. Answer the same question for the regular M/M/1 queue.