Exam Optimization of Business Processes
25 May 2010

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions count equally.
The use of a calculator and a dictionary are allowed.

1a. Give a formula for the expected waiting time in a queue with Poisson arrivals, deterministic service times, FCFS service discipline, and a single server.
b. The same as a but now with service times that can be any of two values, each with a certain probability.
c. The same as b but now with a priority rule depending on the service time duration.
d. Compute the answers to b and c for a non-trivial example (that is, a positive arrival rate, different service time durations, and $\rho < 1$).

2. A medical department has on a certain day of the week a demand for CT scans that is Poisson distributed with an average of 4. Having a patient without a slot is twice as expensive as a leftover slot. How many slots should be reserved?
a. Calculate the expected costs for a few possibilities and calculate the optimal number of slots.
b. Model the problem as an inventory model and verify the results found at a.

3. A contact center has inbound calls and emails. Shifts are defined by 0-1 vectors. There are K different types of shifts, and shift k costs c_k. At interval i s_i agents are needed for inbound calls. During interval i u_i agents are required for dealing with the emails.
a. Formulate a mathematical programming model for shift scheduling during one day that minimizes costs and schedules enough agents (i.e., at least $s_i + u_i$ during interval i).
Now the emails from interval i need not necessarily be handled during interval i, but in one of the intervals $i, \ldots, i + t - 1$ for some fixed $t > 1$.
b. Formulate a mathematical programming model for shift scheduling during one day that minimizes costs and schedules enough agents.
c. Give a simple numerical example in which the answer under b is cheaper than the one under a.

4a. Explain in words the concepts of booking limits and bid prices.
b. Which method can best be used in the case of multiple resources (such as multiple flight legs)? Motivate your answer.
c. Explain the difference between EMSR-a and EMSR-b.

Table with value of P(X>k) with X a Poisson distributed random variable with mean mu

	values of mu									
k	1	2	3	4	5	6	7	8	9	10
0	0.632	0.865	0.950	0.982	0.993	0.998	0.999	1.000	1.000	1.000
1	0.264	0.594	0.801	0.908	0.960	0.983	0.993	0.997	0.999	1.000
2	0.080	0.323	0.577	0.762	0.875	0.938	0.970	0.986	0.994	0.997
3	0.019	0.143	0.353	0.567	0.735	0.849	0.918	0.958	0.979	0.990
4	0.004	0.053	0.185	0.371	0.560	0.715	0.827	0.900	0.945	0.971
5	0.001	0.017	0.084	0.215	0.384	0.554	0.699	0.809	0.884	0.933
6	0.000	0.005	0.034	0.111	0.238	0.394	0.550	0.687	0.793	0.870
7	0.000	0.001	0.012	0.051	0.133	0.256	0.401	0.547	0.676	0.780
8	0.000	0.000	0.004	0.021	0.068	0.153	0.271	0.407	0.544	0.667
9	0.000	0.000	0.001	0.008	0.032	0.084	0.170	0.283	0.413	0.542
10	0.000	0.000	0.000	0.003	0.014	0.043	0.099	0.184	0.294	0.417
11	0.000	0.000	0.000	0.001	0.005	0.020	0.053	0.112	0.197	0.303
12	0.000	0.000	0.000	0.000	0.002	0.009	0.027	0.064	0.124	0.208
13	0.000	0.000	0.000	0.000	0.001	0.004	0.013	0.034	0.074	0.136
14	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.017	0.041	0.083
15	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.008	0.022	0.049
16	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.011	0.027
17	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.014
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.007
19	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002
21	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
22	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000