Exam Optimization of Business Processes
27 May 2009

This exam consists of **4** problems, each consisting of several questions.
All answers should be motivated, including calculations, formulas used, etc.
It is allowed to use 1 sheet of paper (or 2 sheets written on one side) with **hand-written** notes.
The minimal note is 1. All questions count equally.
The use of a calculator and a dictionary are allowed.
A table with the normal distribution is attached.

1. Consider a revenue management model with 2 classes, the Littlewood model. The revenues are: $y_1 = 4$, $y_2 = 1$. The demand of class 1 is Poisson distributed with average 9.
a. What is the optimal amount of capacity that needs to be reserved for class 1? Use a normal approximation for the demand.
It is now possible to use overselling. When a ticket is bought back from a class-2 customer then this costs 1 more.
b. What is the optimal reservation level?
c. Describe a method to compute the expected number of unused capacity for a given reservation level, assuming ample demand of class 2.

2a. Formulate the call center shift scheduling model with K different shifts and a service level constraint for each time interval.
Now assume there is other work, and for each interval an agent can either do inbound calls or the other work. Over the day a total amount of A time intervals have to be dedicated to the other type of work.
b. Extend the shift scheduling model to allow for both types of work.
c. A manager decides that only a subset of the agents need to be trained for the other type of work. Reformulate the model again to account for this situation.

3. Consider a system with 4 components. Components 1 and 2 have an exponential(λ_1) lifetime, components 3 and 4 have an exponential(λ_2) lifetime. Two components need to be up to have a functioning system. The system starts with components 3 and 4 as spare parts.
a. Calculate the expected time that the system is up when components 3 and 4 are in cold stand-by.
b. Calculate the expected time that the system is up when components 3 and 4 are in warm stand-by.
c. Calculate ϕ and Φ for this system.

4. The waiting line for an MRI facility is modeled as an M/M/1 queue, with the additional feature that the arrival rate decreases linearly in the number of waiting patients until it reaches 0.
a. Model this system as a birth-death process.
b. Formulate the stationary probabilities of this process.
c. What is the most likely state in this system, i.e., the state with the highest stationary probability?
d. Answer the same question for the regular M/M/1 queue.

Table with value of $P(0<X<x+y)$ with X a random variable with a standard normal distribution

	values of y									
values of x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.032	0.036
0.1	0.040	0.044	0.048	0.052	0.056	0.060	0.064	0.067	0.071	0.075
0.2	0.079	0.083	0.087	0.091	0.095	0.099	0.103	0.106	0.110	0.114
0.3	0.118	0.122	0.126	0.129	0.133	0.137	0.141	0.144	0.148	0.152
0.4	0.155	0.159	0.163	0.166	0.170	0.174	0.177	0.181	0.184	0.188
0.5	0.191	0.195	0.198	0.202	0.205	0.209	0.212	0.216	0.219	0.222
0.6	0.226	0.229	0.232	0.236	0.239	0.242	0.245	0.249	0.252	0.255
0.7	0.258	0.261	0.264	0.267	0.270	0.273	0.276	0.279	0.282	0.285
0.8	0.288	0.291	0.294	0.297	0.300	0.302	0.305	0.308	0.311	0.313
0.9	0.316	0.319	0.321	0.324	0.326	0.329	0.331	0.334	0.336	0.339
1	0.341	0.344	0.346	0.348	0.351	0.353	0.355	0.358	0.360	0.362
1.1	0.364	0.367	0.369	0.371	0.373	0.375	0.377	0.379	0.381	0.383
1.2	0.385	0.387	0.389	0.391	0.393	0.394	0.396	0.398	0.400	0.401
1.3	0.403	0.405	0.407	0.408	0.410	0.411	0.413	0.415	0.416	0.418
1.4	0.419	0.421	0.422	0.424	0.425	0.426	0.428	0.429	0.431	0.432
1.5	0.433	0.434	0.436	0.437	0.438	0.439	0.441	0.442	0.443	0.444
1.6	0.445	0.446	0.447	0.448	0.449	0.451	0.452	0.453	0.454	0.454
1.7	0.455	0.456	0.457	0.458	0.459	0.460	0.461	0.462	0.462	0.463
1.8	0.464	0.465	0.466	0.466	0.467	0.468	0.469	0.469	0.470	0.471
1.9	0.471	0.472	0.473	0.473	0.474	0.474	0.475	0.476	0.476	0.477
2	0.477	0.478	0.478	0.479	0.479	0.480	0.480	0.481	0.481	0.482
2.1	0.482	0.483	0.483	0.483	0.484	0.484	0.485	0.485	0.485	0.486
2.2	0.486	0.486	0.487	0.487	0.487	0.488	0.488	0.488	0.489	0.489
2.3	0.489	0.490	0.490	0.490	0.490	0.491	0.491	0.491	0.491	0.492
2.4	0.492	0.492	0.492	0.492	0.493	0.493	0.493	0.493	0.493	0.494
2.5	0.494	0.494	0.494	0.494	0.494	0.495	0.495	0.495	0.495	0.495
2.6	0.495	0.495	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
2.7	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497	0.497
2.8	0.497	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.498
2.9	0.498	0.498	0.498	0.498	0.498	0.498	0.498	0.499	0.499	0.499
3	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499