Operations Research – Test 1, 2024 exercises Test 1, 2024

Write your names and student numbers on each sheet that you hand in. Your answer should give a clear explanation, with intermediate steps and calculations included. Good luck!

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1. A company is producing three products (A, B, and C) in the upcoming period. These three products are crafted from the same two raw materials (1 and 2).
   To craft one unit of product A, 10 kg of raw material 1 and 30 kg of raw material 2 are needed. To craft one unit of product B, 40 kg of raw material 1 and 10 kg of raw material 2 are needed. To craft one unit of product C, 20 kg of raw material 1 and 30 kg of raw material 2 are needed. Maximal 50 units of product A can be sold for a price of 300 euros per unit. Maximal 40 units of product B can be sold for a price of 550 euros per unit. Maximal 45 units of product C can be sold for a price of 500 euros per unit.
   There is a regulation that at least 40 % of the produced units should be of product A (the cheapest of the three products). The current purchase prices for raw materials are 11 euros per kg for raw material 1 and 8 euros per kg for raw material 2. There is no limit on the amounts of raw material the company can purchase for these standalone prices. Additionally, the company has an option to purchase raw maw material 1 and raw material 2 together for the same amount of kg. Such a bundle containing equal weights of both raw materials has a 10% discount in price compared to purchasing raw materials 1 and 2 separately for the current standalone prices. The 10% discount in price is offered for bundles which contain at most 1000 kg of each of the two raw materials.

   The company needs to create a production plan for the upcoming period. The objective of the company is to maximize the net profit in the upcoming period through crafting and selling of the three products given the current prices and satisfying the constraints following from the problem description.

   Formulate a linear program (LP) that describes this optimization problem maximizing the total netto profit for the company. Explain the meaning of all decision variables and constraints of your LP formulation. Be careful to (re)write all constraints as a linear (in)equality. Be clear if you make any assumptions or simplifications in your LP formulation.

   You do **not** need to solve the LP you have formulated! The most challenging part of this problem is figuring out what is a good choice of decision variables. Think about this carefully.

2. Solve the following LP using the simplex method. In the first pivot step choose x_1 as basis entering variable. If you then apply the simplex method correct you will need two pivot steps to find the optimal solution.

$$\begin{aligned} \max \quad & z = 5x_1 + 2x_2 - x_3 & \\ \text{subject to} \quad & 4x_1 + 2x_2 + x_3 & \leq 17 \\ & 2x_1 + x_2 - 2x_3 & \leq 6 \\ & x_1, x_2, x_3 \geq 0 & \end{aligned}$$