**Exam Operations Research**

Date: June11, 2020
Time: 10:00 - 12:15

**Points per exercise**:

- Exercise 1 has a total of 25 points (a(10), b(5) and c(10)).
- Exercise 2 has a total of 15 points.
- Exercise 3 has a total of 20 points (a(5), b(5) and c(10)).
- Exercise 4 has a total of 20 points (a(10) and b(10)).
- Exercise 5 has a total of 10 points.

Thus in total 90 points can be obtained. The exam grade is determined as follows:

$$\text{Exam grade } = 1 + \frac{\text{total number of obtained points}}{10}.$$

- Calculator is allowed.

- This exam consists of 5 pages, including this one.

- The duration of this exam is **2 hours and 15 minutes**. When you finish working you notify the host of your meeting. Within 15 minutes after you finish working you have to upload the scanned pdf file in Canvas in the assignment for the June 11 exam. After uploading the pdf file you notify the host again.

- Students who have obtained permission for extra time may use an *additional 30 minutes*.

**Exercise 1**

Consider the following LP which is referred to as the "primal LP".

$$\begin{aligned} \max \quad & z = x_1 + 3x_2 - x_3 + x_4 \\ \text{s.t.} \quad & x_1 + 2x_2 - 2x_3 + x_4 \leq 10 \\ & x_1 + 2x_2 + x_3 = 20 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

(a) **[10 points]** Determine the dual of this LP.

(b) **[5 points]** The primal LP has been solved by the simplex method resulting in the following final tableau:

| Basic | $z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $s_1$ | $r_1$ | value |
|---|---|---|---|---|---|---|---|---|
| $z$ | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 30 |
| $x_4$ | 0 | 3 | 6 | 0 | 1 | 1 | 2 | 50 |
| $x_3$ | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 20 |

Determine the optimal solution of the primal LP and optimal objective value. Determine also the optimal solution of the dual LP and optimal dual objective function.

(c) **[10 points]** The extra constraint $x_4 \leq 20$ is added to the primal LP. Apply the dual simplex method to determine the optimal solution and optimal objective value of the modified LP.
*Instruction: If you do the calculations correctly you get some fractions in the simplex tableau, but all these fractions are of the form $\frac{1}{6}k$ with $k$ integer. Moreover, it will require only one pivot step to find the new optimal solution.*

**Exercise 2**

There are 3 distribution centers ($i = 1, 2, 3$) from which a certain product can be transported to fulfill the weekly demand for this product of 4 towns ($j = 1, 2, 3, 4$). Town $j$ has a weekly demand for this product of $D_j$ tons. The cost of transporting the product from distribution center $i$ to town $j$ is $C_{ij}$ per ton. Moreover, distribution centre $i$ has a startup cost $A_i$ (per week) if this product is transported from distribution centre $i$ to at least one town. In other words the weekly startup cost $A_i$ is not incurred if there is no transport of this product from distribution centre $i$. For each distribution centre $i$ it is not possible to transport more than $B_i$ tons of the product per week.

(a) **[15 points]** Formulate the problem of minimizing the total (of transport and startup) costs per week such that the demand for the product is fulfilled in each town as a mixed integer linear program. Explain the meaning of all variables and constraints you have in your mixed integer linear program formulation.

**Exercise 3**

Consider the following problem. You are packing your suitcase for a journey and need to decide what items to bring. You have 4 items (just one of each); each has a certain value to you, and a certain weight:

| | | | | |
|---|---|---|---|---|
| weight (kg) | 4 | 7 | 8 | 11 |
| value | 8 | 15 | 18 | 24 |

You do not want to have more than 16 kg of weight in the suitcase. The problem is to choose under that weight restriction which items should be put into the suitcase maximizing the total value of those items.

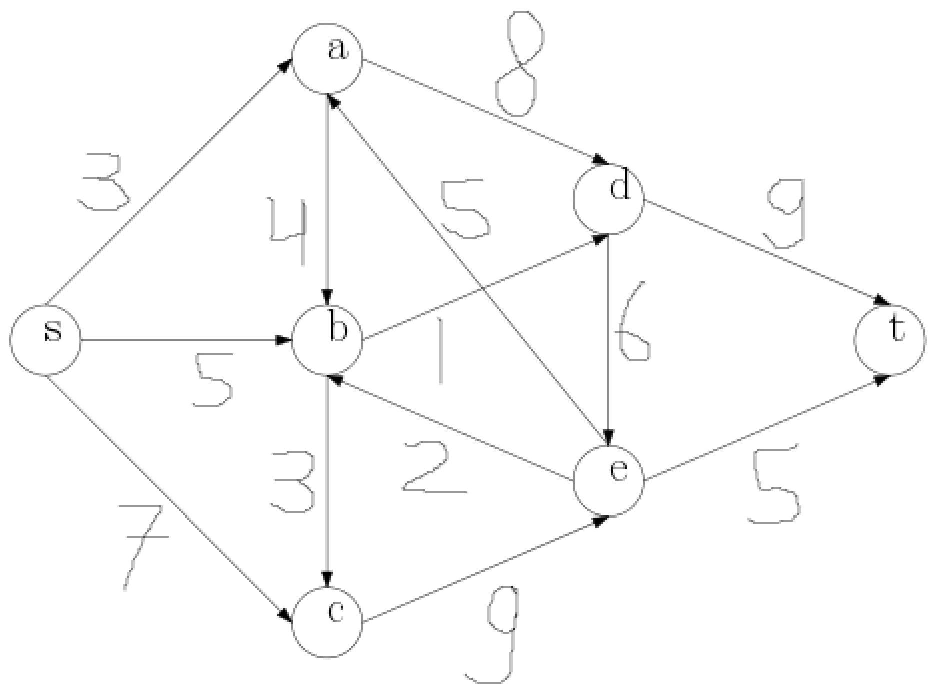
(a) **[5 points]** Formulate an integer linear program for this problem.

(b) **[5 points]** Determine the unique optimal solution of the LP relaxation of this problem. Also determine the corresponding optimal objective value.

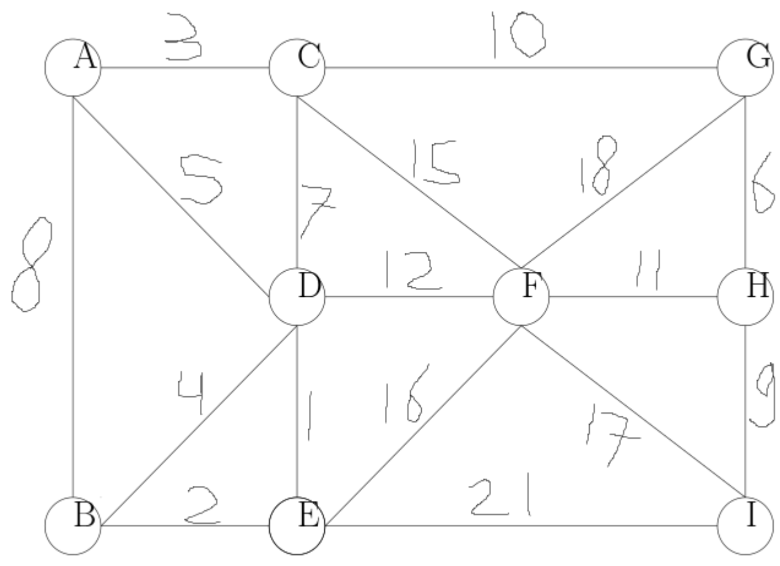
(c) **[10 points]** Solve the ILP from (a) by applying the **branch-and-bound** method. Clearly indicate in which order you compute the nodes of the search tree and where you prune the search tree. Also indicate which pruning criterion you use when you prune.

**Exercise 4**

(a) **[10 points]** Consider the instance of the maximum flow problem shown in the directed graph below where the arc capacities are indicated).
Let the current flow $f$ (which is feasible but not maximal) be as follows: $f_{sa} = 3$, $f_{sb} = 4$, $f_{sc} = 5$, $f_{ad} = 6$, $f_{bd} = 1$, $f_{bc} = 3$, $f_{ce} = 8$, $f_{dt} = 7$, $f_{ea} = 3$, $f_{et} = 5$ and no flow on all other arcs of the given graph. Draw the residual graph $D^f$ corresponding to this flow $f$. Continue from this residual graph the Ford-Fulkerson algorithm to determine a maximum flow from $s$ to $t$. State the value of the flow and show that it is maximal by giving a minimum $s$-$t$ cut of that value.




(b) **[10 points]** Consider the problem of finding a minimum weight spanning tree in the non-directed graph below (where edge weights have been indicated) using Kruskal's algorithm. Make clear in which order the edges are picked by the algorithm and draw the minimum weight spanning tree which is finally obtained. Explain your answer briefly.



**Exercise 5**

Consider the acyclic directed graph shown below with length of the arcs as indicated.

(a) **[10 points]**. Apply dynamic programming to determine the **longest path** from node $A$ to node $K$ in this directed graph. It should be clear that you have applied dynamic programming. At the end present clearly the longest path which you have found.