Exam Operations Research
CALCULATOR IS NOT ALLOWED

Construction of the exam

This exam consists of 6 exercises, from which in total 80 points can be obtained.

Exercise	*a*	*b*	*c*
Exc. 1	5	10	5
Exc. 2	10	-	-
Exc. 3	5	5	5
Exc. 4	10	5	-
Exc. 5	10	-	-
Exc. 6	10	-	-

Exam mark

$$\frac{\text{total number of points}}{8}$$

Final course mark

$$\frac{1}{4}\text{Instruction tests} + \frac{3}{4}\text{Exam mark}$$

Exercise 1
Consider the LP-problem

$$\begin{aligned} \max \quad & 6x_1 - 3x_2 + 5x_3 \\ \text{s.t.} \quad & 4x_1 + 4x_2 + x_3 \leq 20 \\ & x_1 + x_2 + 3x_3 \leq 4 \\ & 2x_1 - x_2 + x_3 \leq 9 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \tag{1}$$

(a) Translate this LP-problem into standard basis form. Give the starting solution for the simplex algorithm and apply *one* iteration to get from the starting solution to an improved basic feasible solution. Expose and argue precisely how you find this improved basic feasible solution.

(b) What are the new basic variables? What are their values and what is the new objective value. How can you read from the new simplex tableau if the new solution is the optimal one? Is this the case here?

(c) Formulate the dual LP-problem. Without solving the dual, give the optimal dual solution and the optimal dual objective value.

Exercise 2. *Please notice that for correct ingredients of the model in this exercise you can already gain points.*
A group of N general physicians (GPs) has decided to collaborate. However, they think it is important that every patient is assigned a GP who is his/her primary contact. They agree to reassign the whole set of M patients. The past records of each patient i give an estimate t_i of the number of hours per year he/she requires consultation. In the reassignment they like to take distance between patient addresses and the GP's location into account: the distance between GP j and the address of patient i is d_{ij} $[i = 1, 2, ..., M; j = 1, ..., N]$. GP j has specified that he prefers to have a set of patients whose estimated consultancy requirements together do not exceed T_j hours per year, $[j = 1, ..., N]$. But patients have some influence. Each patient i has given a list S_i of GPs to whom they do *not* want to be assigned, $[i = 1, ..., M]$. Finally, patients 1 and 2 are a couple who have been promised to become assigned to the same GP. The GPs wish to make an assignment of the patients such that the total distance between the patient's addresses and the GPs assigned to them is minimized.

Formulate this problem as an integer linear programming problem.

Exercise 3.
Consider the following 0-1 knapsack problem.

$$\begin{aligned} \max \quad & 5x_1 + 12x_2 + 9x_3 + 2x_4 \\ \textit{onder}: \quad & 2x_1 + 5x_2 + 4x_3 + 1x_4 \leq 10 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

(a) Give the unique solution of the LP-Relaxation of this problem. Also give the corresponding objective value of the LP-Relaxation.

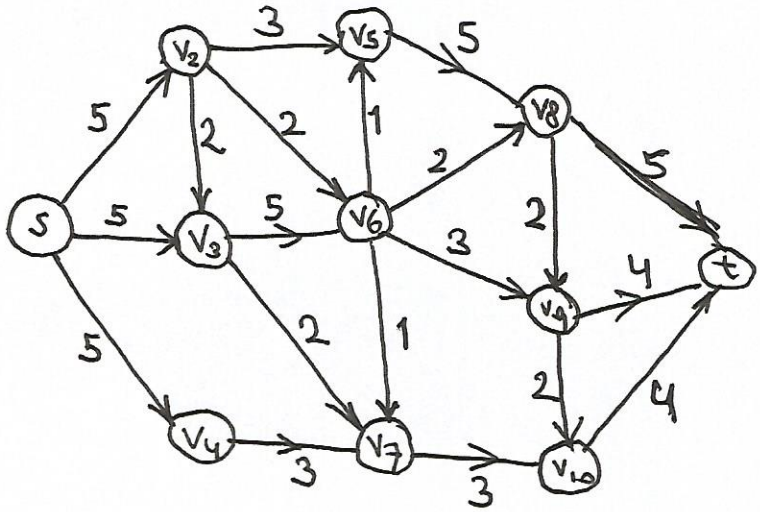
(b) Solve the 0-1 knapsack problem by Branch-and-Bound to find the optimal solution and the corresponding optimal value.

(c) Solve the *Knapsack* problem obtained by replacing the 0-1 constraints on the variables by non-negative integrality constraints.

Exercise 4.
Consider the network in the figure. The number next to an arrow indicates the capacity of that arrow.

(a) Find the maximum flow from s to t using Ford-Fulkerson. Show clearly which flow augmenting paths you choose in each iteration.

(b) Using the residual network in the optimum, find the minimum s-t-cut. Clearly show how you find this minimum cut.



s
v2
v3
v4
v5
v6
v7
v8
v9
v10
t
5
5
5
3
2
2
5
1
2
3
1
2
3
3
5
5
2
4
2
4

Exercise 5.
Delete the directions on the arcs in the figure of **Exercise 4**, making them undirected edges. Interpret the numbers in the figure as the costs of the corresponding edges. Find the Minimum Spanning Tree in the graph using your preferred method.

Exercise 6. *Please notice that for correct ingredients of the model in this exercise you can already gain points.*
Kylie is planning a world-trip and likes to visit a total of N different countries, the countries $1, \ldots, N$, and eventually return to the Netherlands (= country 0). She starts also in the Netherlands and the order in which she will visit the N countries is still to be determined. If Kylie flies from country i to country k it will cost her an amount of v_{ik} euro, $i = 0, \ldots, N,\ k = 0, \ldots, N$. Moreover, she does not want to visit any country twice or more. **Notice**: the $(N+1)$-th country she will visit is fixed, the Netherlands.

How will her optimal world-trip be (including the return to the Netherlands), such as to minimize total travel expenses?

Formulate this problem as a dynamic programming problem. To do so, describe the states and the interpretation of the value function in words. Give the recursion and the starting conditions and describe what needs to be computed in terms of the value function. ***Hint:*** *Start with stage j.*