Exam Operations Research, 30-05-2016
CALCULATOR IS NOT ALLOWED

Construction of the exam

This exam consists of 6 exercises, from which 100 points can be obtained in total. The division of points over the various parts is given in the table.

Exercise	*a*	*b*	*c*
Exc. 1	10	5	10
Exc. 2	10	-	-
Exc. 3	5	10	-
Exc. 4	10	5	-
Exc. 5	15	-	-
Exc. 6	20	-	-

Exam mark

$$\frac{\text{total number of points}}{10}$$

Final course mark

$$\frac{1}{4}\text{Instruction mark} + \frac{3}{4}\text{Exam mark}$$

Exercise 1

$$\begin{aligned} \max \quad & 12x_1 - 6x_2 + 10x_3 \\ \text{s.t.} \quad & 2x_1 + 2x_2 + 1x_3 \leq 5 \\ & 2x_1 + 2x_2 + 6x_3 \leq 8 \\ & 2x_1 - x_2 + x_3 \leq 9 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \tag{1}$$

(a) Translate this LP-problem into standard basis form. Give the starting solution for the simplex algorithm and apply *one* iteration to get from the starting solution to an improved basic feasible solution. Show precisely how you find this improved basic feasible solution.

(b) Give the new basic variables and their values. Is this new basic solution optimal? Explain your answer.

(c) Formulate the dual LP-problem.

Exercise 2 *Please notice that for correct parts of the model in this exercise you can already gain points.*

Every week Manon makes N different hamburger sauces for the company Burger Meister, numbered $j = 1, 2, ..., N$. Each sauce is produced by mixing M ingredients, numbered $i = 1, 2, ..., M$. There is a restriction $N > M$. There is a certain amount of freedom while making the sauces, but the following conditions need to be satisfied: every sauce has to consist of at least 10% of ingredient 1. Sauce 2 has to consist of at least 25% of ingredient 3. Further, Burger Meister wants to deliver four times as much sauce 4 as of sauce 2. Each week Manon can order at most b_i liter of ingredient $i, i = 1, 2, ..., M$. Manon can sell the sauces for a price of w_j $(j = 1, 2, ...N)$ per liter, she can sell as much she produces. Manon can buy ingredient i for c_i euro per liter. Formulate this problem as an integer linear optimization problem.

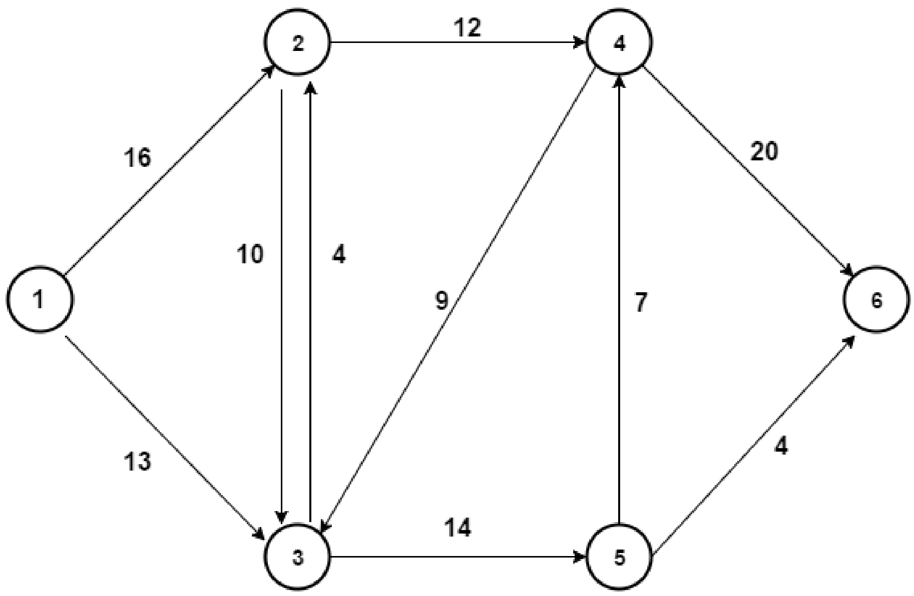
Exercise 3
Consider the following 0-1 knapsack problem.

$$\begin{array}{rl} \max & 5x_1 + 12x_2 + 9x_3 + 2x_4 \\ s.t. & 2x_1 + 5x_2 + 4x_3 + x_4 \leq 10 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{array}$$

(a) Determine the unique solution of the LP-Relaxation of this problem. Also give the corresponding optimal objective value of the LP-Relaxation.

(b) Solve the 0-1 knapsack problem using Branch and Bound. Clearly indicate in which order you compute the nodes of the search tree, where you prune the search tree, and why you prune.

Exercise 4
Consider the max-flow problem on the network in the figure. The number next to an arrow represents the capacity of that arrow.



(a) Determine the maximum flow from 1 to 6 using Ford-Fulkerson. Show clearly which flow augmenting paths you choose in each iteration.

(b) Give the minimum $1-6$-*cut* and its value. Confirm your answer using the maximum flow of **(a).**

Exercise 5
The numbers of the graph in **Exercise 4** now represent the lengths of the corresponding arrows. Find the shortest path of 1 to 6 using Dijkstra's algorithm. Explain your algorithm and clearly state the order of your algorithm.

Exercise 6

The Eredivisie, the Dutch football league, decides on a new set up for the coming year. To make the competition as exciting as possible all players are reassigned to new clubs. The KNVB (the Dutch football association) has been given the task to assign all M players to the N clubs that play in the Eredivisie. Obviously players have varying salaries which clubs will have to pay them. s_i is the salary of player i. At the same time clubs have varying salary budgets, B_j, which is the budget of club j. In general a club can only be assigned players as long as the total sum of the salaries of those players does not exceed the clubs budget.

To assess player strength, every player has been given a value between 0 and 100, where 100 is assigned to the best player in the league. The strength of a player is given by p_i, the strength of player i.

The goal of the assignment is to make sure the combined strength of all players of the worst team is as high as possible.

Formulate this problem as a dynamic programming model. To do so, describe the states and the interpretation of the value function in words. Give the recursion and the starting conditions and describe what needs to be computed in terms of the value function. **Hint: start with the stage.**