Networks and Graphs lecture 7

Tree

> *a **Tree** is a connected, simple and acyclic graph.*

for **connected graphs: $n \leq m + 1$** where n are vertices and m are edges.

> *a connected graph G with **n vertices and n-1 edges** is a **tree**.*
>
> *a simple graph G is a **tree** if and only if for **all u,v there is exactly one (u,v)-path** in G.*
>
> *a connected graph G is a **tree** if and only if **every edge is a cut edge**.*

minimum spanning tree (MST)

> *a **tree $T \subseteq G$ is a spanning tree of G if $V(T) = V(G)$.***
>
> *a spanning tree is **minimal** if the **sum of its weights are minimal**.*

if any edge gets added to a spanning tree a cycle is created.

let G be a simple connected graph and there will be constructed 2 non-empty partitions V_1 and V_2, the edge e of minimum weight must be contained in a tree T (MST).

Kruskal's algorithm

let G be a connected weighted graph, divide all vertices into its own subtrees.

1. select the edge with the lowest weight and connect the nodes(they join the subtrees into one).
2. link subtrees until all vertices are contained in one subtree (repeat step above).
3. don't join 2 vertices contained in the same subtree.

prim-jarnik algorithm

let G be a simple connected weighted graph.

1. select a vertex to start the algorithm.
2. expand the subtree with the lowest weight edge to the next vertex(don't use edges to vertices already in the subset).
3. repeat the step above until a spanning tree is created