Networks and Graphs lecture 6

Chinese postman problem

the goal of the chinese postman problem is to find a minimal closed walk of a graph.

solution:

1. pair every vertex of odd degree to another and contruct a path and duplicate them.
2. the resulting will have an Euler tour.

Hamiltonian graphs

> *a Hamilton cycle contains every vertex exactly once.*
>
> *a graph is Hamiltonian if it has a Hamiltonian cycle.*

brute force: explore all possible paths from a vertex. (very inefficient).

> *if a graph G is Hamiltonian then for any V* subset of V(G) the graph G −V* contains at−most |V*|* **components**.

> *if G has n >= 3 vertices and all vertices have a degree >= n/2, then G is Hamiltonian.*

posa's algorithm

1. select any vertex v
2. select a neighbor of v not part of the path
3. add the neighbor to the path
4. if no new neighbor can be found perform a rotational transform on a neighbor of the last vertex
5. repeat until either a Hamiltonian path is found or all options have been depleted

the algorithm might fail to find a Hamiltonian cycle.

traveling salesman problem

goal of the problem is to find a shortest(minimal weight) hamilton cycle in an complete weighted graph.

greedy algorithm

1. start with an arbitrary cycle
2. swap 2 edges if it reduces the overall weight

hamilton cycles in weighted graphs

a digraph D can be transposed to a graph G

- for each vertex v in D add vertices v*in* and v*out*
- add all edges from the digraph in the particular direction.
- add weights to the edges between the vin and vout vertices.

check for a hamilton cycle in the newly constructed graph G.